CARIBBEAN EXAMINATIONS COUNCIL

# CSEC®
# Additional Mathematics

SYLLABUS
SPECIMEN PAPER
MARK SCHEME
SUBJECT REPORTS

Macmillan Education
4 Crinan Street, London, N1 9XW
A division of Macmillan Publishers Limited
Companies and representatives throughout the world

www.macmillan-caribbean.com

ISBN 978-0-230-48271-5

www.cxc.org
www.cxc-store.com



First published 2014
This revised edition published 2018



Designed by Macmillan Publishers Limited
Cover design by Macmillan Publishers Limited and Red Giraffe

**CSEC® Additional Mathematics Free Resources**

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## Additional Mathematics

This Additional Mathematics course provides a variety of topics with related attributes which would enable Caribbean students to reason logically using the prior knowledge gained from the CSEC General Proficiency Mathematics. Candidates are expected to enter this course of study with a solid foundation of algebraic knowledge and mathematical reasoning.

On completing this course students will be able to make a smooth transition to higher levels of study in Mathematics, or move on to career choices where a deeper knowledge of the general concepts of Mathematics is required. This course of study, which includes fundamentals of Pure and Applied Mathematics, caters to diverse interests enabling students to develop critical-thinking skills applicable to other subject areas. This course thus provides insight into the exciting world of advanced mathematics, thereby equipping students with the tools necessary to approach any mathematical situation with confidence.

The syllabus is arranged as a set of topics, arranged in four sections as follows:

- Section 1 Algebra and Functions
- Section 2 Coordinate Geometry and Trigonometry
- Section 3 Introductory Calculus
- Section 4 Basic Mathematical Applications

# CARIBBEAN EXAMINATIONS COUNCIL

**Caribbean Secondary Education Certificate**
**CSEC®**

# ADDITIONAL MATHEMATICS SYLLABUS

Effective for examinations from May/June 2012

Published by the Caribbean Examinations Council



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# Contents

# Additional Mathematics Syllabus

## ♦ RATIONALE

The Caribbean, with few limited resources, has prided itself in being a knowledge-based society utilizing the skills and assets of people, our greatest resource, to progress in a dynamic world where self-reliance is now more than ever a needed goal. Although different languages are spoken in the Caribbean, the language of Mathematics is one of the forms in which people of the Caribbean effectively communicate with each other.

This Additional Mathematics course provides a variety of topics with related attributes which would enable Caribbean students to reason logically using the prior knowledge gained from the CSEC General Proficiency Mathematics. Candidates are expected to enter this course of study with a solid foundation of algebraic knowledge and mathematical reasoning.

On completing this course students will be able to make a smooth transition to higher levels of study in Mathematics, or move on to career choices where a deeper knowledge of the general concepts of Mathematics is required. This course of study, which includes fundamentals of Pure and Applied Mathematics, caters to diverse interests enabling students to develop critical-thinking skills applicable to other subject areas.

Some of the underlying concepts of Mathematics will be explored to a level which fosters a deeper understanding and greater appreciation of Mathematics. This will give students the confidence and ability to approach problem-solving in enlightened ways and lead to creative methods of solving complex real-world problems.

This course thus provides insight into the exciting world of advanced mathematics, thereby equipping students with the tools necessary to approach any mathematical situation with confidence.

*This syllabus will contribute to the development of the Ideal Caribbean Person as articulated by the CARICOM Heads of Government in the following areas: "demonstrate multiple literacies, independent and critical thinking and innovative application of science and technology to problem solving. Such a person should also demonstrate a positive work attitude and value and display creative imagination and entrepreneurship". In keeping with the UNESCO Pillars of Learning, on completion of this course the study, students will learn to do, learn to be and learn to transform themselves and society.*

## ♦ AIMS

The syllabus aims to:

1. build upon those foundational concepts, techniques and skills acquired at the CSEC General Proficiency Level and form linkages to areas of study at the Advanced Proficiency Level;

2. enhance ways of learning Mathematics;

3. stimulate further curiosity and analytical thinking in deriving solutions to problems which may not necessarily be solved by a single/unique approach;

4. promote effective mathematical communication;

5. develop abilities to reason logically;

6. develop skills in formulating real-world problems into mathematical form;

7. develop positive intrinsic mathematical values, such as, accuracy and rigour;

8. connect Mathematics with other disciplines such as Science, Business and the Arts.

## ♦ PRE-REQUISITES OF THE SYLLABUS

Any person with a good grasp of the contents of the syllabus of the Caribbean Secondary Education Certificate (CSEC) General Proficiency course in Mathematics, or equivalent, should be able to undertake this course. However, successful participation in this course will also depend critically on the possession of good verbal and written communication skills.

## ♦ ORGANIZATION OF THE SYLLABUS

The syllabus is arranged as a set of topics, and each topic is defined by its specific objectives and content. It is expected that students would be able to master the specific objectives and related content after successfully pursuing a course in Mathematics during five years of secondary education.

The topics are arranged in four sections as follows:

Section 1 - Algebra and Functions

Section 2 - Coordinate Geometry and Trigonometry

Section 3 - Introductory Calculus

Section 4 - Basic Mathematical Applications

## ♦ SUGGESTIONS FOR TEACHING THE SYLLABUS

For students who complete CSEC Mathematics in the fourth form year, Additional Mathematics can be done in the fifth form year. Alternatively students may begin Additional Mathematics in the fourth form and sit both CSEC Mathematics and Additional Mathematics examinations at the end of form five. Students may even do the CSEC Additional Mathematics as an extra subject simultaneously with CAPE Unit 1 in the Sixth Form.

## ♦ CERTIFICATION AND DEFINITION OF PROFILES

The syllabus will be examined for certification at the General Proficiency Level.

In addition to the overall grade, there will be a profile report on the candidate's performance under the following headings:

(i) Conceptual Knowledge (CK);
(ii) Algorithmic Knowledge (AK);
(iii) Reasoning (R).

The assessment will test candidates' skills and abilities in relation to three cognitive levels.

Conceptual knowledge - the ability to **recall**, **select** and **use** appropriate facts, concepts and principles in a variety of contexts.

Algorithmic knowledge - the ability to **manipulate** mathematical expressions and procedures using appropriate symbols and language, logical deduction and inferences.

Reasoning - the ability to **select**, **use** and **evaluate** mathematical models and **interpret** the results of a mathematical solution in terms of a given real-world problem, and to **engage** problem-solving.

## ♦ FORMAT OF THE EXAMINATIONS

The examination will consist of three papers: Paper 01, an objective type paper, Paper 02, an essay or problem-solving type paper and Paper 03, the School Based Assessment which will focus on investigation or problem solving related to any area of the syllabus.

**Paper 01**
(1 hour 30 minutes)

This Paper will consist of 45 multiple-choice items, sampling the Core as follows:

| Section | Topics | No. of items | Total |
|---|---|---|---|
| 1 | Algebraic Operations | 2 | 20 |
| | Quadratics | 3 | |
| | Inequalities | 2 | |
| | Functions | 4 | |
| | Surds, Indices and Logarithms | 5 | |
| | Series | 4 | |
| 2 | Co-ordinate Geometry | 3 | 14 |
| | Vectors | 3 | |
| | Trigonometry | 8 | |
| 3 | Differentiation | 5 | 11 |
| | Integration | 6 | |
| | **Total** | | **45** |

The 45 marks will be weighted to 60 marks

**Paper 02** (2 hours 40 minutes)

This Paper will consist of two sections, Section I and Section II.

Section I: 80 marks — This section will consist of 6 compulsory structured and problem-solving type questions based on Sections 1, 2 and 3 of the syllabus: Algebra and Functions; Coordinate Geometry, Vectors and Trigonometry; and Introductory Calculus.

Section II: 20 marks — This section will consist of 2 structured or problem-solving questions based on Section 4 of the syllabus, Basic Mathematical Applications. One question will be set on Data Representation and Probability and the other question will be set on Kinematics.
Candidates will be required to answer only **ONE** question from this section Each question will be allocated 20 marks.

The marks allocated to the sections are shown below.

| Sections | | No. of questions | Marks | | | Total |
|---|---|---|---|---|---|---|
| | | | **CK** | **AK** | **R** | |
| 1 | Algebra and Functions | 2 | 6 | 12 | 10 | 28 |
| 2 | Coordinate Geometry, Vectors and Trigonometry | 2 | 6 | 10 | 8 | 24 |
| 3 | Introductory Calculus | 2 | 6 | 12 | 10 | 28 |
| 4 | Basic Mathematical Applications | 1 of 2 | 6 | 8 | 6 | 20 |
| **Total Marks** | | | 24 | 42 | 34 | 100 |

**SCHOOL BASED ASSESSMENT**

**Paper 03/1**

This paper comprises a project requiring candidates to demonstrate the practical application of Mathematics in everyday life. In essence it should allow candidates to probe, describe and explain a mathematical area of interest and communicate the findings using mathematical symbols, language and tools. The topic(s) chosen may be from any section or combination of different sections of the syllabus.

The project may require candidates to collect data, or may be theory based, requiring solution or proof of a chosen problem.

The total marks for Paper 03/1 is 20 and will contribute 20% toward the final assessment. See Guidelines for School Based Assessment on pages 29 – 46.

**Paper 03/2 (Alternative to Paper 03/1), examined externally.**

This paper is an alternative to Paper 03/1 and is intended for private candidates. This paper comprises one question. The given topic(s) may be from any section or combination of different sections of the syllabus. The duration of the paper is 1 ½ hours.

**WEIGHTING OF PAPER AND PROFILES**

The percentage weighting of the examination components and profiles is as follows:

**Table 1 – Percentage Weighting of Papers and Profiles**

| PROFILES | PAPER 01 | PAPER 02 | PAPER 03 | TOTAL % |
|---|---|---|---|---|
| Conceptual (CK) | 12 (16) | 24 | 04 (08) | 47 (24%) |
| Algorithmic Knowledge (AK) | 24 (32) | 42 | 06 (12) | 87 (44%) |
| Reasoning (R) | 09 (12) | 34 | 10 (20) | 66 (32%) |
| TOTAL | 45 (60) | 100 | 20 (40) | 200 |
| [%] | 30% | 50% | 20% | 100% |

## ♦ REGULATIONS FOR RESIT CANDIDATES

1. Resit candidates must complete Papers 01 and 02 of the examination for the year for which they re-register. Resit candidates who have earned at least 50% of the **MODERATED** score for the SBA component may elect not to repeat this component, provided they re-write the examination no later than the year following their first attempt. The scores for the SBA can be transferred once only, that is, to the examination immediately following that for which they were obtained.

2. Resit candidates who have obtained less than 50% of the **MODERATED** scores for the SBA component must repeat the component at any subsequent sitting.

3. Resit candidates must be entered through a school or other approved educational institution.

## ♦ REGULATIONS FOR PRIVATE CANDIDATES

Candidates who are registered privately will be required to sit Paper 01, Paper 02 and Paper 03/2.

Private candidates must be entered through institutions recognized by the Council.

## ♦ MISCELLANEOUS SYMBOLS

| | |
|---|---|
| $=$ | is equal to |
| $\neq$ | is not equal to |
| $<$ | is less than |
| $\leq$ | is less than or equal to (is not greater than) |
| $>$ | is greater than |
| $\geq$ | is greater than or equal to (is not less than) |
| $\equiv$ | is identical to |
| $\approx$ | is approximately equal to |
| $\propto$ | is proportional to |
| $\infty$ | infinity |

**Operations**

| | |
|---|---|
| $\sum_{i=1}^{n} x_i$ | $x_1 + x_2 + x_3 + \ldots + x_n$ |

**Functions**

| | |
|---|---|
| P | the set of Real Numbers |
| f ($x$) | the value of the function f at $x$ |
| $\mathrm{f}^{-1}$ | the inverse function of the function f |
| g * f, gf | the composite function f and g which is defined by $(\mathrm{g} * \mathrm{f})(x)$ or $\mathrm{gf}(x) = \mathrm{g}[\mathrm{f}(x)]$ |
| $\frac{\mathrm{d}y}{\mathrm{d}x}$, $y'$ | the first derivative of $y$ with respect to $x$ |
| $\frac{\mathrm{d}^n y}{\mathrm{d}x^n}$, $y^n$ | the $n^{\text{th}}$ derivative of $y$ with respect to $x$ |
| $\mathrm{f}'(x), \mathrm{f}''(x), \cdots, \mathrm{f}^{(n)}(x)$ | the first, second, …, $n^{\text{th}}$ derivatives of f($x$) with respect to $x$ |
| $\dot{x}, \ddot{x}$ | the first and second derivatives of $x$ with respect to time |
| lg $x$ | the logarithm of $x$ to base 10 |
| $\int y \, \mathrm{d}x$ | the indefinite integral of $y$ with respect to $x$ |
| $\int_a^b y \, \mathrm{d}x$ | the definite integral of $y$ with respect to $x$ between the limits $x = a$ and $x = b$ |

**Probability and Statistics**

| | |
|---|---|
| $A \cap B$ | union of the events $A$ and $B$ |
| $A \cup B$ | intersection of the events $A$ and $B$ |
| $S$ | the possibility sample space |
| $P(A)$ | the probability of the event $A$ occurring |
| $P(A')$ | the probability of the event $A$ not occurring |
| $P(A \mid B)$ | the conditional probability of the event $A$ occurring given the event $B$ has occurred. |

**Vectors**

| | |
|---|---|
| $\underset{\sim}{a}$, **a** | the vector **a** |
| $\overrightarrow{AB}$ | the vector represented in magnitude and direction by the directed line segment AB |
| $\lvert\overrightarrow{AB}\rvert$, | the magnitude of $\overrightarrow{AB}$ |
| $\hat{\mathbf{a}}$ | a unit vector in the direction of **a** |
| $\lvert\mathbf{a}\rvert$ | the magnitude of **a** |
| **a.b** | the scalar (dot) product of **a** and **b** |
| **i**, **j** | unit vectors in the direction of the Cartesian coordinate axes, $x$ and $y$ respectively |
| $\begin{pmatrix} x \\ y \end{pmatrix}$ | $x\mathbf{i} + y\mathbf{j}$ |

**Mechanics**

| | |
|---|---|
| $x$ | displacement |
| $v$, $\dot{x}$ | velocity |
| $a$, $\dot{v}$, $\ddot{x}$ | acceleration |
| $g$ | acceleration due to gravity |

## ♦ LIST OF FORMULAE

Arithmetic Series

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}[2a + (n-1)d]$$

Geometric Series

$$T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r - 1} \qquad S_\infty = \frac{a}{1-r}, \quad -1 < r < 1 \text{ or } |r| < 1$$

Circle: $x^2 + y^2 + 2fx + 2gy + c = 0 \qquad (x + f)^2 + (y + g)^2 = r^2$

Vectors

$$\hat{\mathbf{v}} = \frac{\mathbf{v}}{|\mathbf{v}|} \qquad \cos\theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| \times |\mathbf{b}|} \qquad |\mathbf{v}| = \sqrt{(x^2 + y^2)} \text{ where } \mathbf{v} = x\mathbf{i} + y\mathbf{j}$$

Trigonometry

$$\sin(A \pm B) \equiv \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) \equiv \cos A \cos B \mp \sin A \sin B$$

Differentiation

$$\frac{\mathrm{d}}{\mathrm{d}x}(ax + b)^n = an(ax + b)^{n-1}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\sin x = \cos x$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\cos x = -\sin x$$

Statistics

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i} \qquad S^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n} = \frac{\sum_{i=1}^{n} f_i x_i^2}{\sum_{i=1}^{n} f_i} - (\bar{x})^2$$

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Kinematics

$$v = u + at \qquad v^2 = u^2 + 2as \qquad s = ut + \frac{1}{2}at^2$$

## ♦ USE OF ELECTRONIC CALCULATORS

**Candidates are expected to have an electronic non-programmable calculator and are encouraged to use such a calculator in Paper 02. Candidates will also be allowed to use a calculator in Papers 01 and 03.**

Guidelines for the use of electronic calculators are listed below.

1. Silent, electronic hand-held calculators may be used.
2. Calculators should be battery or solar powered.
3. Candidates are responsible for ensuring that calculators are in working condition.
4. Candidates are permitted to bring a set of spare batteries in the examination room.
5. **No** compensation will be given to candidates because of faulty calculators.
6. **No** help or advice is permitted on the use or repair of calculators during the examination.
7. Sharing calculators is **not** permitted in the examination room.
8. Instruction manuals, and external storage media (for example, card, tape, disk, smartcard or plug-in modules) are **not** permitted in the examination room.
9. Calculators with graphical display, data bank, dictionary or language translation are **not** allowed.
10. Calculators that have the capability of communication with any agency in or outside of the examination room **are prohibited**.

## ♦ SECTION 1 : ALGEBRA AND FUNCTIONS

### GENERAL OBJECTIVES

On completion of this Section, students should:

1. be confident in the manipulation of algebraic expressions and the solutions of equations and inequalities;

2. understand the difference between a sequence and a series;

3. distinguish between convergence and divergence of arithmetic and geometric series;

4. understand the concept of a function;

5. develop the ability to use concepts to model and solve real-world problems.

| SPECIFIC OBJECTIVES | CONTENT |
|---|---|
| **A. Algebra** | |
| Students should be able to: | |
| 1. perform operations of addition, subtraction, multiplication and division of polynomial and rational expressions; | Addition, subtraction, multiplication, division and factorization of algebraic expressions. |
| 2. factorize polynomial expressions, of degree less than or equal to 4, leading to real linear factors; | Division of a polynomial of degree less than or equal to 4 by a linear or quadratic polynomial. |
| 3. apply the Remainder Theorem; | Remainder Theorem. |
| 4. use the Factor Theorem to find factors and to evaluate unknown coefficients. | Factor Theorem. |
| **B. Quadratics** | |
| Students should be able to: | |
| 1. express the quadratic function $ax^2 + bx + c = 0$ in the form | Quadratic equations in one unknown. |

## ALGEBRA AND FUNCTIONS (cont'd)

| SPECIFIC OBJECTIVES | CONTENT |
|---|---|
| Students should be able to: | |
| $a(x+h)^2 + k = 0$, where $h$ and $k$ are constants to be determined. | Completing the square. |
| 2. determine maximum or minimum values and range of quadratic functions by completion of the square; | |
| 3. sketch the graph of the quadratic function, including maximum or minimum points; | Graphs of quadratic functions. |
| 4. determine the nature of the roots of a quadratic equation; | |
| 5. solve equations in $x$ reducible to a quadratic equation, for example, $x^4 - 6x^2 + 8 = 0$ and $x - 2\sqrt{x} + 1 = 0$ ; | |
| 6. use the relationship between the sums and products of the roots and the coefficients of $ax^2 + bx + c = 0$; | Applications of sums and products of the roots of quadratic equations. |
| 7. solve two simultaneous equations in 2 unknowns in which one equation is linear and the other equation is quadratic. | Solution of equations (one linear and one quadratic). |
| **C. Inequalities** | |
| Students should be able to: | |
| 1. find the solution sets of quadratic inequalities using algebraic and graphical methods; | Quadratic inequalities in one unknown. |
| 2. find the solution sets of inequalities | Rational inequalities with linear factors. |

| SPECIFIC OBJECTIVES | CONTENT |
|---|---|
| of the form $\frac{ax+b}{cx+d} > 0; \geq 0; < 0; \leq 0$ using algebraic and graphical methods. | |
| **D. Functions** | |
| Students should be able to: | |
| 1. use terms related to functions; | Arrow diagrams. Function, domain, co-domain, range, open interval, half open interval, closed interval, one-to-one function, onto function, one-to-one correspondence, inverse and composition of functions; |
| 2. determine the range of a function given its domain; | Rational and polynomial functions up to degree less than or equal to 3. |
| 3. determine whether a given function is many-to-one or one-to-one; | Graphical methods and horizontal line test. Formal proof not required. |
| 4. determine the inverse of a given function, (if it exists); | |
| 5. plot and sketch functions and their inverses, (if they exist); | Exclude rational functions. |
| 6. state the geometrical relationship between the function $y$ = f($x$) and its inverse $f^{-1}(x)$; | $f^{-1}(x)$ as the reflection of f ($x$) in the line $y = x$. |
| 7. find the composition of two functions; | |
| 8. recognize that, if g is the inverse of f, then f [g($x$)] = $x$, for all $x$, in the domain of g. | |
| **E. Surds, Indices, and Logarithms** | |
| Students should be able to: | |
| 1. perform operations involving surds; | Addition, subtraction, multiplication and rationalization of denominators of surds. |

**SPECIFIC OBJECTIVES** | **CONTENT**

Students should be able to:

2. use the laws of indices to solve exponential equations with one unknown;

Equations reducible to linear and quadratic forms.

3. use the fact that $\log_a b = c \Leftrightarrow a^c = b$ where *a* is any positive whole number;

The relationship between indices and logarithms.

4. simplify expressions by using *the laws*:

Laws of logarithms.

(i) $\log_a (PQ) = \log_a P + \log_a Q$;

(ii) $\log_a \left(\frac{P}{Q}\right) = \log_a P - \log_a Q$;

(iii) $\log_a P^b = b \log_a P$;

(iv) $\log_a a = 1$;

(v) $\log_a 1 = 0$;

5. solve logarithmic equations;

Example,

$$\log_a (2x + 5) - \log_a (3x - 10) = \log_a (x - 14).$$

6. use logarithms to solve equations of the form $a^x = b$;

7. apply logarithms to problems involving the transformation of a given relationship to linear form.

Linear Reduction.

**F. Sequences and Series**

Students should be able to:

1. define a sequence of terms $\{a_n\}$ where *n* is a positive integer;

2. write a specific term from the formula for the $n^{th}$ term of a sequence;

## ALGEBRA AND FUNCTIONS (cont'd)

| SPECIFIC OBJECTIVES | CONTENT |
| --- | --- |
| 3. use the summation $(\sum)$ notation; | |
| 4. define a series, as the sum of the terms of a sequence; | Series as the sum of the terms of a sequence. |
| 5. identify arithmetic and geometric series; | |

Students should be able to:

| | |
| --- | --- |
| 6. obtain expressions for the general terms and sums of finite arithmetic and finite and infinite geometric series; | The sums of finite arithmetic, and finite and infinite geometric series. |
| 7. show that all arithmetic series (except for zero common difference) are divergent, and that geometric series are convergent only if $-1 < r < 1$, where $r$ is the common ratio; | |
| 8. calculate the sum of arithmetic series to a given number of terms; | |
| 9. calculate the sum of geometric series to a given number of terms; | |
| 10. find the sum of a convergent geometric series. | |

### Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives of this Section, teachers are advised to engage students in the teaching and learning activities listed below.

**Number Systems**

Teachers should do a brief review of the Number Systems before starting Algebra and Functions.

**Functions (one-to-one, onto, one-to-one correspondence) – Inverse Function**

Students should explore the mapping properties of quadratic functions which:

(i) will, or will not, be one-to-one, depending on which subset of the real line is chosen as the domain;

(ii) will be onto, if the range is taken as the co-domain (completion of the square is useful here);

(iii) if both one-to-one and onto, will have an inverse function which can be obtained by solving a quadratic equation.

<u>Example</u>: Use the function f : A → B given by $f(x) = 3x^2 + 6x + 5$, where the domain A is alternatively the whole of the real line, or the set $\{x \in \mathbf{R} \mid x \geq -1\}$, and the co-domain B is **R** or the set $\{y \in \mathbf{R} \mid y \geq 2\}$.

**Series**

Teachers should apply the concepts of the arithmetic and geometric series to solve real-world problems such as investments.

## ♦ SECTION 2: COORDINATE GEOMETRY, VECTORS AND TRIGONOMETRY

### GENERAL OBJECTIVES

On completion of this Section, students should:

1. develop the ability to represent and deal with points in the coordinate plane through the use of geometry and vectors;
2. develop the ability to manipulate and describe the behaviour of trigonometric functions;
3. develop skills to solve trigonometric equations;
4. develop skills to prove simple trigonometric identities;
5. develop the ability to use concepts to model and solve real-world problems.

| SPECIFIC OBJECTIVES | CONTENT |
|---|---|
| **A. Coordinate Geometry** | |
| Students should be able to: | |
| 1. find the equation of a straight line; | The gradient of a line segment. |
| 2. determine whether lines are parallel or mutually perpendicular using the gradients; | Relationships between the gradients of parallel and mutually perpendicular lines. |
| 3. find the point of intersection of two lines; | |
| 4. write the equation of a circle; | The equation of the circle in the forms $(x+f)^2+(y+g)^2=r^2$, $x^2+y^2+2fx+2gy+c=0$, where $a, b, f, g, c, r \in \mathrm{P}$. |
| 5. find the centre and radius of a given circle; | |
| 6. find equations of tangents and normals at given points on circles; | Tangents and normals to the circle. |
| 7. find the points of intersection of a curve with a straight line. | |

| SPECIFIC OBJECTIVES | CONTENT |
|---|---|
| **B. Vectors** | |
| Students should be able to: | |
| 1. express a vector in the form $\begin{pmatrix} x \\ y \end{pmatrix}$ or $x\mathbf{i} + y\mathbf{j}$; $x, y \in \text{P}$; | Two-dimensional vectors and their geometric representations. |
| 2. define equal vectors; | Equality of vectors. |
| 3. add and subtract vectors; | |
| 4. multiply a vector by a scalar quantity; | |
| 5. derive and use unit vectors; | Unit vectors. |
| 6. find displacement vectors; | Position and displacement vectors. |
| 7. find the magnitude and direction of a vector; | Modulus and direction of a vector. |
| 8. define the scalar product of two vectors:<br>(i) in terms of their components;<br>(ii) in terms of their magnitudes and the angle between them; | Scalar (dot) product of 2 vectors. |
| 9. find the angle between two given vectors; | |
| 10. apply properties of parallel and perpendicular vectors. | Problems involving parallel and perpendicular vectors. |
| **C. Trigonometry** | |
| (*All angles will be assumed to be measured in radians unless otherwise stated*) | |
| 1. define the radian; | |
| 2. convert degrees to radians and radians to degrees; | |
| 3. use the formulae for arc length $l = rq$ and sector area $A = \frac{1}{2} r^2 q$; | Applications of arc length and sector area. |

| SPECIFIC OBJECTIVES | CONTENT |
|---|---|
| Students should be able to: | |
| 4. evaluate sine, cosine and tangent for angles of any size given either in degrees or radians; | |
| 5. evaluate the exact values of sine, cosine and tangent for $q = 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$, , , $2\pi$; | Include related angles such as $\frac{2\pi}{3}, \frac{4\pi}{3}$. |
| 6. graph the functions sin *kx*, cos *kx*, tan *kx*, where *k* is 1 or 2 and $0 \leq x \leq 2\pi$; | |
| 7. derive the identity $\cos^2 \theta + \sin^2 \theta \equiv 1$; | |
| 8. use the formulae for sin ($A \pm B$), cos ($A \pm B$) and tan ($A \pm B$); | Compound-angle formulae. |
| 9. derive the multiple angle identities for sin 2*x*, cos 2*x*, tan 2*x*; | Double-angle formulae. |
| 10. use Specific Objectives 7, 8 and 9 above to prove simple identities; | |
| 11. find solutions of simple trigonometric equations for a given range, including those involving the use of $\cos^2 q + \sin^2 q \equiv 1$. | Solution of simple trigonometric equations including graphical interpretation but excluding general solution. |

**Suggested Teaching and Learning Activities**

To facilitate students' attainment of the objectives of this Section, teachers are advised to engage students in the teaching and learning activities listed below.

**Trigonometric Identities**

Teachers should derive the trigonometric identities and formulae where appropriate. However, students are not expected to know the proofs of the following trigonometric formulae:

sin ($A \pm B$), cos ($A \pm B$), tan ($A \pm B$).

Students should recognise that $\tan\theta \equiv \dfrac{\sin\theta}{\cos\theta}$ .

Students should use the equilateral and the isosceles right angled triangle to derive the exact values of sine, cosine and tangent of $\left(\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}\right)$.

Students should also derive the trigonometric functions sin $x$ and cos $x$ for angles $x$ of any value (including negative values), using the coordinates of points on the unit circle.

Students should be made aware of the relationships between the unit circle and its quadrants and the related angles (principal and secondary solutions).

## ♦ SECTION 3: INTRODUCTORY CALCULUS

### GENERAL OBJECTIVES

On completion of this Section, students should:

1. understand the relationships between the derivative of a function at a point and the behaviour of the function and its tangent at that point;
2. be confident in differentiating and integrating given functions;
3. understand the relationship between integration and differentiation;
4. know how to calculate areas and volumes using integration;
5. develop the ability to use concepts to model and solve real-world problems.

| SPECIFIC OBJECTIVES | CONTENT |
|---|---|
| **A. Differentiation** | |
| Students should be able to: | |
| 1. use the concept of the derivative at a point $x = c$ as the gradient of the tangent to the graph at $x = c$; | The gradient of a curve. |
| 2. define the derivative at a point as a limit; | The derivative as a limit (intuitive approach). |
| 3. use the $f'(x)$ and $\frac{dy}{dx}$ notation for the first derivative of f ($x$); | |
| 4. use $\frac{d}{dx} x^n = n x^{n-1}$ where $n$ is any real number; | The derivative of $x^n$. |
| 5. use $\frac{d}{dx} \sin x = \cos x$ and $\frac{d}{dx} \cos x = - \sin x$; | The derivatives of $\sin x$ and $\cos x$. |

## SECTION 3: INTRODUCTORY CALCULUS (cont'd)

| SPECIFIC OBJECTIVES | CONTENT |
|---|---|
| Students should be able to: | |
| 6. use simple rules of derivatives to find derivatives of sums and multiples of functions; | Simple rules of derivatives: (i) $\frac{d}{dx}c\,f(x) = c\frac{d}{dx}f(x)$ where $c$ is a constant (ii) $\frac{d}{dx}f(x) \pm g(x) = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x)$ |
| 7. use Specific Objectives 4, 5 and 6 above to calculate derivatives of polynomials and trigonometric functions; | Differentiation of simple polynomials and trigonometric functions involving sine and cosine only. |
| 8. apply the chain rule in the differentiation of composite functions; | Function of a function, the chain rule. |
| 9. differentiate products and quotients of simple polynomials and trigonometric functions; | Product and quotient rules. |
| 10. use the concept of the derivative as a rate of change; | |
| 11. use the concept of stationary points; | Stationary points. |
| 12. determine the nature of stationary points; | |
| 13. locate stationary points, maxima and minima, by considering sign changes of the derivative; | Point(s) of inflexion not included. |
| 14. calculate the second derivative, $f''(x)$; | Second derivatives of functions. |
| 15. interpret the significance of the sign of the second derivative; | |
| 16. use the sign of the second derivative to determine the nature of stationary points; | |
| 17. obtain equations of tangents and normals to curves. | |

## SECTION 3: INTRODUCTORY CALCULUS (cont'd)

| SPECIFIC OBJECTIVES | CONTENT |
|---|---|
| **B. Integration** | |
| Students should be able to: | |
| 1. recognize integration as the reverse process of differentiation; | Anti-derivatives. |
| 2. use the notation $\int f(x)\, dx$; | Indefinite integrals (concept and use). |
| 3. show that the indefinite integral represents a family of functions which differ by constants; | |
| 4. use simple rules of integration; | Rules of Integration.<br>(i) $\int cf(x)\ dx = c\int f(x)\ dx$, where c is a constant;<br>(ii) $\int \{f(x) \pm g(x)\}\, dx = \int f(x)\ dx \pm \int g(x)\ dx$. |
| 5. integrate functions of the form $(ax \pm b)^n$ where *a*, *b*, *n* are real and $n \neq -1$; | Integration of polynomials. |
| 6. find indefinite integrals using formulae and integration theorems; | |
| 7. integrate simple trigonometric functions; | Integration of *a* sin *x* ± *b* cos *x*, where *a* and *b* are constants. |
| 8. compute definite integrals; | The definite integral:<br>$\int_a^b f(x)\, dx$ = F (*b*) - F (*a*), where F (*x*) is an indefinite integral of f (*x*). |
| 9. formulate the equation of a curve given its gradient function and a point on the curve; | |
| 10. apply integration to:<br>(i) find the area of the region in the first quadrant bounded by a curve and the lines parallel to the *y*-axis; | |

| SPECIFIC OBJECTIVES | CONTENT |
|---|---|
| Students should be able to: | |
| (ii) find volumes of revolution about the $x$-axis, for polynomials up to and including degree 2. | The region of the curve to be rotated must be in the first quadrant only. |

**Suggested Teaching and Learning Activities**

To facilitate students' attainment of the objectives of this Section, teachers are advised to engage students in the teaching and learning activities listed below.

**Differentiation**

Teachers should introduce the derivative as a limit but take only an intuitive approach at this stage using diagrams and not first principles.

A graphical explanation of $\frac{\mathrm{d}}{\mathrm{d}x}(\sin x) = \cos x$ would suffice.

Teachers should explain the concept of increasing and decreasing functions, but it will not be tested.

Curve sketching using the differential calculus is not required.

**The Area under the Graph of a Continuous Function**

Class discussion should play a major role in dealing with this topic. Activities such as that which follows may be performed to motivate the discussion.

Example of classroom activity:

Consider a triangle of area equal to $\frac{1}{2}$ unit$^2$, bounded by the graphs of $y = x$, $y = 0$ and $x = 1$.

(i) Sketch the graphs and identify the triangular region enclosed.

(ii) Subdivide the interval [0, 1] into $n$ equal subintervals.

(iii) Evaluate the sum, $s(n)$, of the areas of the inscribed rectangles and $S(n)$, of the circumscribed rectangles, erected on each subinterval.

(iv) By using different values of $n$, for example $n = 5, 10, 15, 20$, show that both $s(n)$ and $S(n)$ get closer to the area of the given region.

## ♦ SECTION 4: BASIC MATHEMATICAL APPLICATIONS

### GENERAL OBJECTIVES

On completion of this Section, students should:

1. appreciate that data can be represented both graphically and numerically to initiate analysis;
2. understand the concept of probability;
3. appreciate that probability models can be used to describe real world situations;
4. apply mathematical models to the motion of a particle.

| SPECIFIC OBJECTIVES | CONTENT |
|---|---|
| **A. Data Representation and Analysis** | |
| Students should be able to: | |
| 1. distinguish between types of data; | Qualitative and quantitative data, discrete and continuous data. |
| 2. represent numerical data diagrammatically; | Stem-and-leaf diagrams and box-and-whisker plots. |
| 3. outline the relative advantages and disadvantages of stem-and-leaf diagrams and box-and-whisker plots in data analyses; | |
| 4. interpret stem-and-leaf diagrams and box-and-whiskers plots; | |
| 5. determine quartiles and percentiles from raw data, grouped data, stem-and-leaf diagrams, box-and-whisker plots; | Percentiles. |
| 6. calculate measures of central tendency and dispersion; | Mode, mean, median, range, interquartile range, semi-inter-quartile range, variance and standard deviation of ungrouped and grouped data; |
| 7. explain how the standard deviation measures the spread of a set of data. | |

## SECTION 4: BASIC MATHEMATICAL APPLICATIONS (cont'd)

| SPECIFIC OBJECTIVES | CONTENT |
|---|---|
| **B. Probability Theory** | |
| Students should be able to: | |
| 1. distinguish among the terms experiment, outcome, sample space and event; | Concept of probability. |
| 2. calculate the probability of event *A*, P(*A*), as the number of outcomes of *A* divided by the total number of possible outcomes, when all outcomes are equally likely and the sample space is finite; | Classical probability.<br><br>Relative frequency as an estimate of probability. |
| 3. use the basic laws of probability:<br>(i) the sum of the probabilities of all the outcomes in a sample space is equal to one;<br>(ii) 0 £ P (*A*) £ 1 for any event *A*;<br>(iii) P (*A*') = 1 – P (*A*), where P (*A*') is the probability that event *A* does not occur; | |
| 4. use P (*A* È *B*) = P(*A*) + P(*B*) - P(*A* Ç *B*) to calculate probabilities; | The addition rule. |
| 5. identify mutually exclusive events *A* and *B* such that P (*A* Ç *B*) = 0; | Mutually exclusive events. |
| 6. calculate the conditional probability P(*A*\|*B*) where $P(A\|B) = \frac{P(A \cap B)}{P(B)}$; | Conditional probability. |
| 7. identify independent events; | Independent events. |
| 8. use the property P(*A* Ç *B*)= P (*A*) × P (*B*) or P(*A*/*B*) = P(*A*) where *A* and *B* are independent events; | |
| 9. construct and use possibility space diagrams, tree diagrams and Venn diagrams to solve problems involving probability. | Possibility space diagrams, tree diagrams and Venn diagrams. |

| SPECIFIC OBJECTIVES | CONTENT |
|---|---|
| **C. Kinematics of Motion along a straight line** | |
| Students should be able to: | |
| 1. distinguish between distance and displacement, and speed and velocity; | Scalar and vector quantities. |
| 2. draw and use displacement-time and velocity-time graphs; | Displacement-time graphs and velocity-time graphs. |
| 3. calculate and use displacement, velocity, acceleration and time in simple equations representing the motion of a particle in a straight line; | Displacement, velocity and acceleration. |
| 4. apply where appropriate the following rates of change: $v = \frac{dx}{dt} = \dot{x}$; $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = \ddot{x}$ where $x, \dot{x}, \ddot{x}$ represent displacement, velocity and acceleration respectively (restricted to calculus from Section 3). | Variable motion of a particle. |

__Suggested Teaching and Learning Activities__

To facilitate students' attainment of the objectives of this Section, teachers are advised to engage students in the teaching and learning activities listed below. Whenever possible, class discussions and presentations should be encouraged.

**Probability**

Consider the three scenarios given below.

1. Throw two dice. Find the probability that the sum of the dots on the uppermost faces of the dice is 6.

2. An insurance salesman visits a household. What is the probability that he will be successful in selling a policy?

3. A hurricane is situated 500 km east of Barbados. What is the probability that it will hit the island?

These three scenarios are very different for the calculation of probability. In '1', the probability is calculated as the number of successful outcomes divided by the total possible number of outcomes. In this classical approach, the probability assignments are based on equally likely outcomes and the entire sample space is known from the start.

The situation in '2' is no longer as well determined as in '1'. It is necessary to obtain historical data for the salesman in question and estimate the required probability by dividing the number of successful sales by the total number of households visited. This frequency approach still relies on the existence of data and its applications are more realistic than those of the classical methodology.

For '3' it is very unclear that a probability can be assigned. Historical data is most likely unavailable or insufficient for the frequency approach. The statistician might have to revert to informed educated guesses. This is quite permissible and reflects the analyst's prior opinion. This approach lends itself to a Bayesian methodology.

One should note that the rules of probability theory remain the same regardless of the method used to estimate the probability of events.

## KINEMATICS

### Definitions

**Displacement** is the position of a point relative to a fixed origin O. It is a vector. The SI Unit is the metre (m). Other metric units are centimeter (cm), kilometer (km).

**Velocity** is the rate of change of displacement with respect to time. It is a **vector**. The SI Unit is **metre per second** ($ms^{-1}$). Other metric units are $cms^{-1}$, $kmh^{-1}$.

**Speed** is the magnitude of the velocity and is a scalar quantity.

**Uniform velocity** is the constant speed in a fixed direction.

**Average velocity** = $\frac{\text{change in displacement}}{\text{time taken}}$

**Average speed** = $\frac{\text{total distance travelled}}{\text{time taken}}$

**Acceleration** is the rate of change of velocity with respect to time. It is a **vector**. The SI Unit is **metre per second square** ($ms^{-2}$). Other metric units are $cms^{-2}$, $kmh^{-2}$.

**Negative acceleration** is also referred to as retardation.

**Uniform acceleration** is the constant acceleration in a fixed direction.

**Motion in one dimension** – When a particle moves in **one dimension**, that is, along a straight line, it has only two possible directions in which to move. Positive and negative signs are used to identify the two directions.

**Vertical motion under gravity** – this is a special case of uniform acceleration in a straight line. The body is thrown **vertically upward**, or falling **freely downward**. This uniform acceleration is due to **gravity** and acts vertically downwards towards the centre of the earth. It is denoted by **g** and may be approximated by **9.8** $\mathbf{ms^{-2}}$.

**GRAPHS IN KINEMATICS**

A **displacement-time** graph for a body moving in a straight line shows its displacement *x* from a fixed point on the line plotted against time, *t*. The **velocity** *v* of the body at time, *t* is given by the **gradient** of the graph since $\frac{dx}{dt} = v$.

The **displacement-time** graph for a body moving with **constant velocity** is a **straight line**. The velocity, *v* of the body is given by the gradient of the line.

The **displacement-time** graph for a body moving with **variable velocity** is a **curve**.
The velocity at any time, *t* may be estimated from the gradient of the tangent to the curve at that time. The average velocity between two times may be estimated from the gradient of the chord joining them.

**Velocity-time graph** for a body moving in a straight line shows its velocity *v* plotted against time, *t*.

The **acceleration,** *a* of a body at time, *t* is given by the **gradient** of the graph at *t*, since $a = \frac{dv}{dt}$.

The **displacement** in a time interval is given by the **area** under the **velocity-time** graph for that time interval.

The **velocity-time** graph for a body moving with **uniform acceleration** is a **straight line**. The acceleration of the body is given by the gradient of the line.

## ♦ GUIDELINES FOR THE SCHOOL BASED ASSESSMENT

**RATIONALE**

School Based Assessment is an integral part of student assessment in the course covered by this syllabus. It is intended to assist students in acquiring certain knowledge, skills and attitudes that are associated with the subject. The activities for the School Based Assessment are linked to the syllabus and should form part of the learning activities to enable the student to achieve the objectives of the syllabus.

During the course of study for the subject, students obtain marks for the competence they develop and demonstrate in undertaking their School Based Assessment assignments. These marks contribute to the final marks and grades that are awarded to students for their performance in the examination.

The guidelines provided in this syllabus for selecting appropriate tasks are intended to assist teachers and students in selecting assignments that are valid for the purpose of School Based Assessment. The guidelines provided for the assessment of these assignments are intended to assist teachers in awarding marks that are reliable estimates of the achievement of students in the School Based Assessment component of the course. In order to ensure that the scores awarded by teachers are in line with the CXC standards, the Council undertakes the moderation of a sample of the School Based Assessment assignments marked by each teacher.

School Based Assessment provides an opportunity to individualise a part of the curriculum to meet the needs of students. It facilitates feedback to the student at various stages of the experience. This helps to build the self-confidence of students as they proceed with their studies. School Based Assessment also facilitates the development of the critical skills and abilities that are emphasised by this CSEC subject and enhances the validity of the examination on which candidate performance is reported. School Based Assessment, therefore, makes a significant and unique contribution to the development of relevant skills and the testing and rewarding of students for the development of those skills.

The Caribbean Examinations Council seeks to ensure that the School Based Assessment scores are valid and reliable estimates of accomplishment. The guidelines provided in this syllabus are intended to assist in doing so.

**Assignment**

The School Based Assessment consists of one project to be marked by the teacher in accordance with CXC guidelines.

There are two types of project.

**Project A** is based on applying mathematical concepts, skills and procedures from any topic (s) in order to understand, describe or explain a real world phenomenon. The project is theory based and no data collection is required.

**Project B** is based on applying mathematical concepts, skills and procedures from any topic (s) in order to understand, describe or explain a real world phenomenon. The project is experiment based and involves the collection of data.

**Candidates should complete <u>one</u> project, either Project A or Project B.**

**<u>Role of the Teacher</u>**

The role of teacher is to:

(i) Suggest the project for the School Based Assessment.

(ii) provide guidance throughout the life of the projects. The teacher should work with candidates to develop a project management chart with definite time lines for achieving clearly identified objectives, from project inception to project completion.

(iii) guide the candidate through the SBA by helping to clarify the problem or by discussing possible approaches to solving the problem. Teachers, while giving guidance, should guard against providing a complete solution to the problem for the candidate or prescribing a specific format that should be followed.

(iv) ensure that the project is developed as a continuous exercise that occurs during scheduled class hours as well as outside class times.

(v) at a time to be determined by the teacher the relevant component will be assessed and the marks recorded. Hardcopies of the completed documents should be kept by both the teacher and student. The teacher should use the mark scheme provided by CXC and include any comments pertinent to the conduct of the assessment.

## ♦ ASSESSMENT CRITRERIA

**Candidates should complete one project, either Project A or Project B.**

**Project A**

The following are the guidelines for assessing this project.

1. Each candidate pursuing Additional Mathematics can complete a project which will be based on applying the mathematical concepts, skills and procedures from any topic (s) contained in any of the sections or their combinations in order to understand, describe or explain a real world phenomenon.

   The project will be presented in the form of a report and will have the following parts.

   (i) A statement of the problem

   A real world problem in Mathematics chosen from any subject or discipline such as Science, Business or the Arts. The student must solve the problem using Specific Objectives completed in the course. This solution will involve either a proof or direct application of the concepts. (Data collection is not usually required for this project. Any necessary data should be given in the problem statement.)

   (ii) Identification of important elements of the problem.

   (iii) Mathematical Formulation of the problem.

   (iv) Selection of a method to solve the problem. This should involve use of Specific Objectives.

   (v) Solution of the resulting mathematical problem.

   (vi) Interpretation of the solution in related real world context.

   (vii) Conclusions reached.

2. The project will be graded out of a total of 20 marks.

   (i) Clarity of the title of the real world problem being studied.

   (ii) Scope/purpose of the problem.

   (iii) Mathematical formulation of the problem.

   (iv) The problem solution including explanation at each step.

   (v) Applications of the solution or proof to the given real world problem.

(vi) Discussion of findings and conclusion statement (this should include suggestions for further analysis of the topic).

(vii) Presentation (including an optional oral question and answer time with the teacher).

**Assessing Project A**

The project will be graded out a total of 20 marks and marks will be allocated to each task as outlined below.

Project Descriptors

1. **Project Title**

- Title is clear and concise, and relates to real world problem (1) [1 ]

2. **Purpose of Project/Problem Statement**

- Purpose is clearly stated and is appropriate in level of difficulty (1) [1 ]

3. **Mathematical Formulation**

- Identifies all the important elements of the problem and shows complete understanding of the relationships among elements (1) [4]
- Shows complete understanding of the problem's mathematical concepts and principles (1)
- Uses appropriate mathematical terminology and notations to model the problem mathematically (1)
- Uses appropriate Mathematical model/methods chosen (1)

4. **The Problem Solution**

- Assumptions are clearly stated (1) [7]
- Proofs are well established (1)
- Diagrams are appropriate and clearly labelled (1)
- Explanations are sufficient and clearly expressed (1)
- Theorems are appropriate and Formulae are relevant to the solution (1)
- Calculations are precise without errors (1)
- Solution is clearly stated (1)

5. **Application of Solution**

- Applies the solution or proof to the given real world problem (1) [2]
- Shows that solution of proof to given problem is valid (1)

6. **Discussion of Findings/Conclusion**

- Discussion is worthwhile (1)
- Conclusion is valid and useful (1) [3]
- Suggestions for future analysis in related areas are incorporated. (1)

7. **Overall Presentation**

[2]

- Presentation is clear and communicates information in a logical way using correct grammar, mathematical jargon and symbols. (2)
- Communicates information in a logical way using correct grammar, mathematical jargon and symbols some of the time. (1)

Total 20 marks

**PROJECT A - EXEMPLAR**

**Title: Use of a Parabolic Reflector in Making a Torch Light.**

**Problem**: A physicist wants to make an instrument that will reflect rays of light from a minute light source in a **vertical beam parallel** to the axis of symmetry as shown in Figure 1.

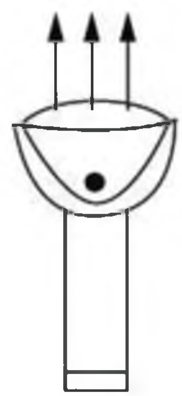


**Figure 1: Physicist's Instrument**

Two crucial decisions he must make concern:

(i) The shape of the reflector to be used.

(ii) The position of the light source.

A. Explain why the physicist should use a parabolic reflector with equation $y = \frac{x^2}{4p}$ and position his light source at the Focus (0, *p*).

B. If the physicist uses the parabolic reflector, as shown in Figure 2, how far from the vertex should the light source be placed on the $y$ axis to produce a beam of parallel rays?

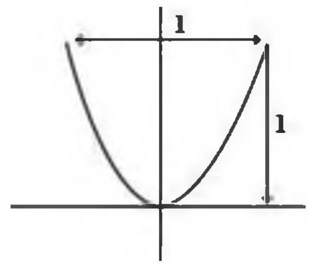



**Figure 2: Parabolic Reflector**

(Assume that when light is reflected from a point $P$, the angle $\alpha$ between an incoming ray and the tangent line at $P$ equals the angle $\beta$ between the outgoing ray and the tangent line at $P$.)

**Mathematical Formulation:**

**A** 1. Show that if we have the distance $FP$ is equal to $PD$ then the equation of the curve

is $y = \dfrac{x^2}{4p}$ (see Figure 3).

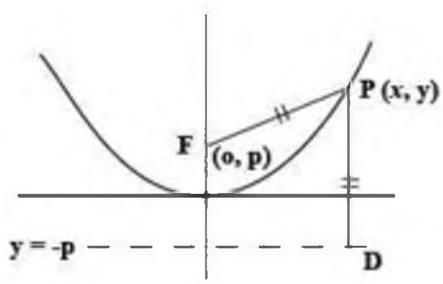



**Figure 3**

2. Show that the tangent line at $P_0$ intersects the $y$ axis at $Q\ (0, -y_0)$

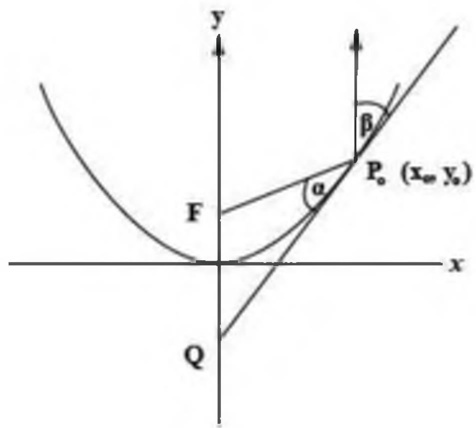



**Figure 4**

3. Prove that the triangle whose vertices are $Q$, $F$, $P_0$ is isosceles.

4. Show that $\alpha = \beta$ in Figure 4.

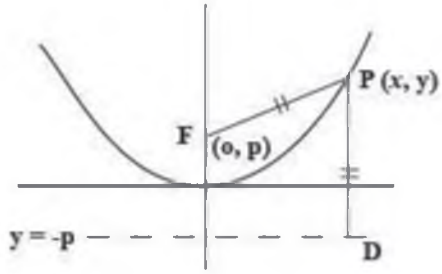
5. Deduce that the physicist should use a parabolic reflector with equation $y = \dfrac{x^2}{4p}$ and position his light source at the Focus (0, $p$). for his instrument to work.

6. Apply to the specific example given in Figure 2.

**Solution of Problem**

A. To show that the physicist should use a parabolic reflector with equation $y = \dfrac{x^2}{4p}$ and position his light source at the focus, F (0, $p$).

1. Show that if for any point $P(x, y)$ on the curve, the distance $FP$ is equal to $PD$, then the equation of the curve is $y = \dfrac{x^2}{4p}$ where F is the point (0, $p$).




Show that $y = \dfrac{x^2}{4p}$ if FP = PD

**PROOF**

**FP=** $\sqrt{(x-o)^2 + (y-p)^2}$

PD = $\sqrt{(y--p)^2}$

$\therefore\ \sqrt{x^2 + (y+p)^2} = \sqrt{(y+p)^2}$

$\therefore\ x^2 + (y - p)^2 = (y + p)^2$ (squaring both sides)

$\therefore\ x^2 + y^2 + p^2 - 2yp = y^2 + p^2 + 2yp$

$\therefore\ x^2 = 4yp, \ \therefore\ y = \dfrac{x^2}{4p}$

The tangent line to the curve, $y = \dfrac{x^2}{4p}$, at the point $P_0$ is shown in Figure 4. Show that the tangent line at $P_0$ intersects the $y$-axis at Q $(0, -y_0)$ .

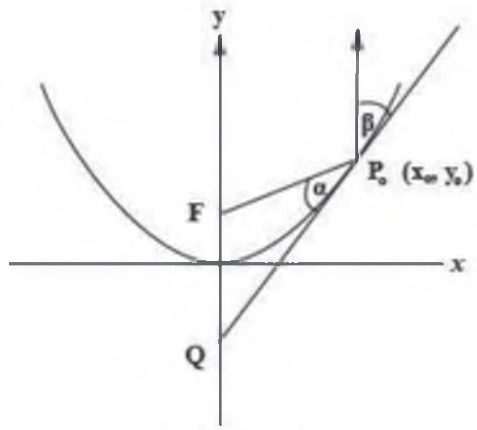



**Figure 4**

The tangent line at $P_0$ intersects the $y$-axis at Q(0, $-y_0$)

**PROOF**

We find the equation of the tangent at $P_0$:

$$\frac{dy}{dx} = \frac{2x}{4p} = \frac{x}{2p}$$

$$\left.\frac{dy}{dx}\right|_{x=x_0} = \frac{x_0}{2p} = \text{slope of tangent.}$$

Since ($x_0$, $y_0$) is a point on the parabola $y_0 = \frac{x_0^2}{4p}$

The equation of the tangent line through ($x_0$, $\frac{x_0^2}{4p}$) is therefore:

$$y - \frac{x_0^2}{4p} = \frac{x_0}{2p}(x - x_0) \quad \text{(i)}$$

To find where this line intersects the $y$-axis we substitute $x = 0$ into (i)

$$\therefore\ y - \frac{x_0^2}{4p} = \frac{x_0}{2p}(-x_0)$$

$$\therefore\ y = \frac{x_0^2}{4p} - \frac{x_0^2}{2p} = \frac{-x_0^2}{4p} = -y_0$$

$\therefore$ The co-ordinates of Q is (0, $-y_o$)

2. Show that the triangle whose vertices are $Q$, $F$, $P_0$ is isosceles.

To show that $\Delta$ $Q$, $F$, $P_0$ is an isosceles triangle, we show that $FQ = FP_0$

**PROOF**

FQ= $\sqrt{(p - -y)^2}$ = $\sqrt{(p+y_0)^2}$ = $p + y_o$

$FP_0$ = $\sqrt{(x_0 - 0)^2 + (y_0 - p)^2}$ = $\sqrt{(x_0^2 + (y_0 - p)^2}$

$F\ P_o$ = $\sqrt{4py_o + \ (y_o - p)^2}$

= $\sqrt{4py_o + \ y_o^2 - 2py_o + p^2}$

= $\sqrt{y_0^2 + 2py_o + p^2}$ = $\sqrt{(y_0 + p)^2}$ = $y_o$ + $p$

∴ $FQ = FP_0$ and Δ $QFP_o$ is isosceles

3. Show that that $\alpha = \beta$.

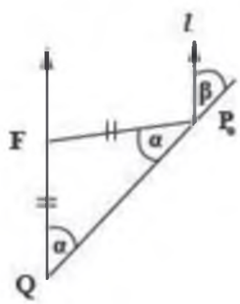



**Figure 5**

Since the base angles of an isosceles Δ are equal $F\hat{Q}P_o = FP_oQ = \alpha$.

But $\alpha$ and $\beta$ are corresponding angles since the line through $P_o$ is parallel to the $y$-axis, therefore, $\alpha = \beta$.

4. Show that the physicist should use a parabolic reflector with equation $y = \dfrac{x^2}{4p}$ and position his light source at the focus, F (0, $p$).

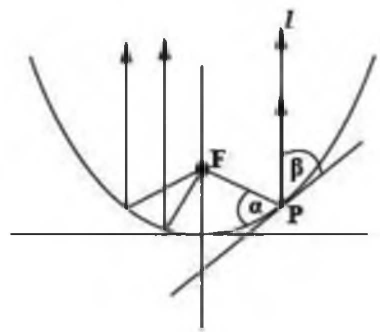



**Figure 6**

The physicist knows that for any ray from *F* striking the reflector at *P*, $\alpha = \beta$
(assumption: angle of incidence equals angle of reflection).

But from part earlier we know that for the parabola when $\alpha = \beta$, the line *l* will be parallel to the axis of symmetry if *F* is (0, *p*).

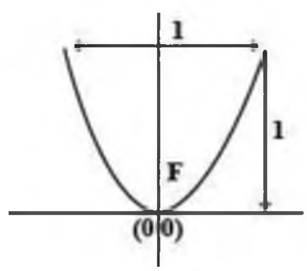
Therefore all rays from the minute light source at *F* that are incident on the parabola will be reflected in a direction parallel to the y axis.

<u>Solution to Part A</u>

Thus the physicist should use a parabolic reflector $y = \frac{x^2}{4p}$ with light source at focus, *F*, to produce a beam of parallel rays.

**Application of the Solution**

B. How far from the vertex should the light source be placed on the *y*-axis to produce a beam of parallel rays?




For the given parabola *y* = 1 when *x* = 1/2

Since y = $\frac{x^2}{4p}$,

$$1 = \frac{\left(\frac{1}{2}\right)^2}{4p}$$

$4p = 1/4$,

$p = \dfrac{1}{16}$

The light source should be placed $\dfrac{1}{16}$ units from the vertex in order to produce a beam of parallel rays.

**Discussion of Findings**

To make an instrument that would reflect a minute light source in a vertical beam parallel to the axis of the symmetry, we have shown that if the physicist uses a parabolic reflector with equation $y = \dfrac{x^2}{4p}$ and positions his light source at the focus, F(0, $p$) then his instrument will work and rays of light will be reflected in a vertical beam.

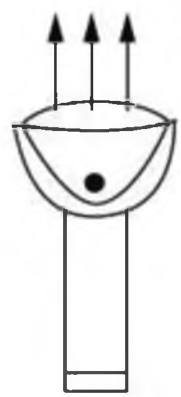


**Figure 1**

For example if parabolic reflection has the dimensions given in Figure 2, he should position his light source at $\dfrac{1}{16}$ units from the vertex of the parabola. This is useful in making torch lights.

**Conclusion and Suggestions**

By understanding the optical properties of a parabola the physicist will know where to position his light source.

It would also be interesting to investigate the optical properties of other curves such as the ellipse, circle and hyperbola to see how their properties can be used to help construct instrutment such as reflecting headlights, telescopes and microscopes.

**Project B**

The following are guidelines for assessing this project.

1. Each candidate pursuing Additional Mathematics can complete a project which will be based on applying the mathematical concepts, skills and procedures from any topic(s) in order to understand, describe or explain a real world phenomenon. This project is experiment based and involves the collection of data.

   The project will be presented in the form of a report and will have the following parts:

   (i) A statement of the problem. A real world problem in Mathematics chosen from any subject or discipline such as Science, Business or the Arts. The student must solve the problem using Specific Objectives completed in the course. This solution will involve data collection required for this project.

   (ii) Identification of important elements of the problem.

   (iii) Formulation of a systematic strategy for representing the problem

   (iv) Data collection appropriate for solving the problem.

   (v) An analysis of the data, information and measurements.

   (vi) Solution of the resulting mathematical problem.

   (vii) Conclusions reached.

2. The project will be graded out of a total of 20 marks:

   (i) Clarity of the title of the real world problem being studied.

   (ii) Scope/purpose of the problem.

   (iii) Method of data collection.

   (iv) Presentation of data.

   (v) Mathematical knowledge/analysis of data.

   (vi) Discussion of findings/conclusions.

   (vii) Presentation.

**ASSESSING PROJECT B**

The project will be graded out a total of 20 marks and marks will be allocated to each task as outlined below.

Project Descriptors

| | | | |
|---|---|---|---|
| 1. | **Project Title** | | [1 ] |
| | • Titled is clear and concise, and relates to real world problem | (1) | |
| 2. | **Purpose of Project** | | [2 ] |
| | • Purpose is clearly stated and is appropriate in level of difficulty | (1) | |
| | • Appropriate variables identified | (1) | |
| 3. | **Method of Data Collection** | | [2 ] |
| | • Data collection method clearly described | (1) | |
| | • Data collection method is appropriate and without flaws | (1) | |
| 4. | **Presentation of Data** | | [3] |
| | • At least one table and one graph/chart used | (1) | |
| | • Data clearly written, labelled, unambiguous and systematic | (1) | |
| | • Graphs, figures, tables and statistical/mathematical symbols used appropriately | (1) | |
| 5. | **Mathematical Knowledge/Analysis of Data** | | [5] |
| | • Appropriate use of mathematical concepts demonstrated | (1) | |
| | • Accurate use of mathematical concepts demonstrated | (1) | |
| | • Some analysis attempted | (1) | |
| | • Analysis is coherent | (1) | |
| | • Analysis used a variety (two or more) of approaches | (1) | |
| 6. | **Discussion of Findings/Conclusion** | | [5] |
| | • Statement of most findings are clearly identified | (1) | |
| | • Statement follows from data gathered/solution of problem | (1) | |
| | • Conclusion based on findings and related to purposes of project | (1) | |
| | • Conclusion is valid | (1) | |
| | • Suggestions for future analysis in related areas | (1) | |
| 7. | **Overall Presentation** | | [2] |
| | • Communicates information in a logical way using correct grammar, | (2) | |

mathematical jargon and symbols most of the time

- Communicates information in a logical way using correct grammar, mathematical jargon and symbols some of the time (1)

Total 20 marks

**PROJECT B – EXEMPLAR**

**Title**

**Simple experiments to determine the fairness of an ordinary game die.**

Statement of Task

Classical probability states that the probability of any of the 6 faces of an ordinary cubical game die landing with a distinct face uppermost after being thrown is $\frac{1}{6}$. It is not unusual for one throwing an ordinary gaming die to observe that one particular face lands uppermost with more frequency than any of the other faces.

Is this sufficient reason for one to conclude that the die may be biased? It may be by chance that this phenomenon occurs, or, perhaps the manner in which the die is thrown has an effect on its outcome. An experiment of this nature may be affected by factors that vary because of the non-uniform manner in which it is conducted.

This project aims to carry out some simple experiments to determine whether or not some varying factors of the manner in throwing the die do in fact influence the outcomes.

Data Collection

An ordinary 6-face gaming die was chosen for this experiment. 120 throws were made for each experiment, using each of the following methods:

(i) holding the die in the palm of the hand and shaking it around a few times before throwing it onto a varnished table top;

(ii) placing the die in a Styrofoam drinking cup, shaking it around a few times before throwing it onto a varnished table top;

(iii) placing the die in a smooth metal drinking cup, shaking it around a few times before throwing it onto a varnished table top;

(iv) holding the die in the palm of the hand and shaking it around a few times before throwing it onto a linen covered table top;

(v) placing the die in a Styrofoam drinking cup, shaking it around a few times before throwing it onto a linen covered table top;

(vi) placing the die in a smooth metal drinking cup, shaking it around a few times before throwing it onto a linen covered table top;

After each experiment the frequencies of the numbers landing uppermost were recorded in tabular form.

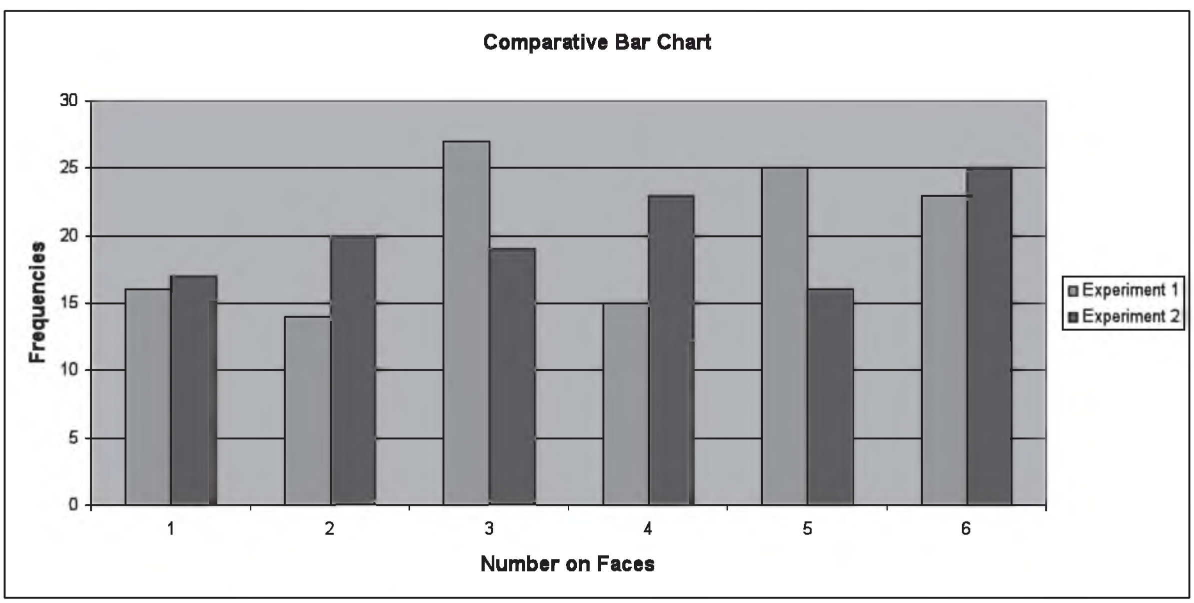
In each of these experiments the number of times the die was shaken before throwing was not predetermined, nor was any other deliberate consideration applied in the subsequent throws. Every effort was taken to avoid bias in each of the experiments.

The following table shows the results of the experiments carried out.

| # on face | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Frequencies – Exp (i) | 16 | 14 | 27 | 15 | 25 | 23 |
| Frequencies – Exp (ii) | 17 | 20 | 19 | 23 | 16 | 25 |
| Frequencies – Exp (iii) | 18 | 25 | 20 | 19 | 25 | 13 |
| Frequencies – Exp (iv) | 16 | 21 | 20 | 29 | 13 | 21 |
| Frequencies – Exp (v) | 13 | 20 | 27 | 18 | 19 | 23 |
| Frequencies – Exp (vi) | 14 | 24 | 17 | 24 | 25 | 16 |
| Total frequencies | 94 | 124 | 130 | 128 | 123 | 121 |

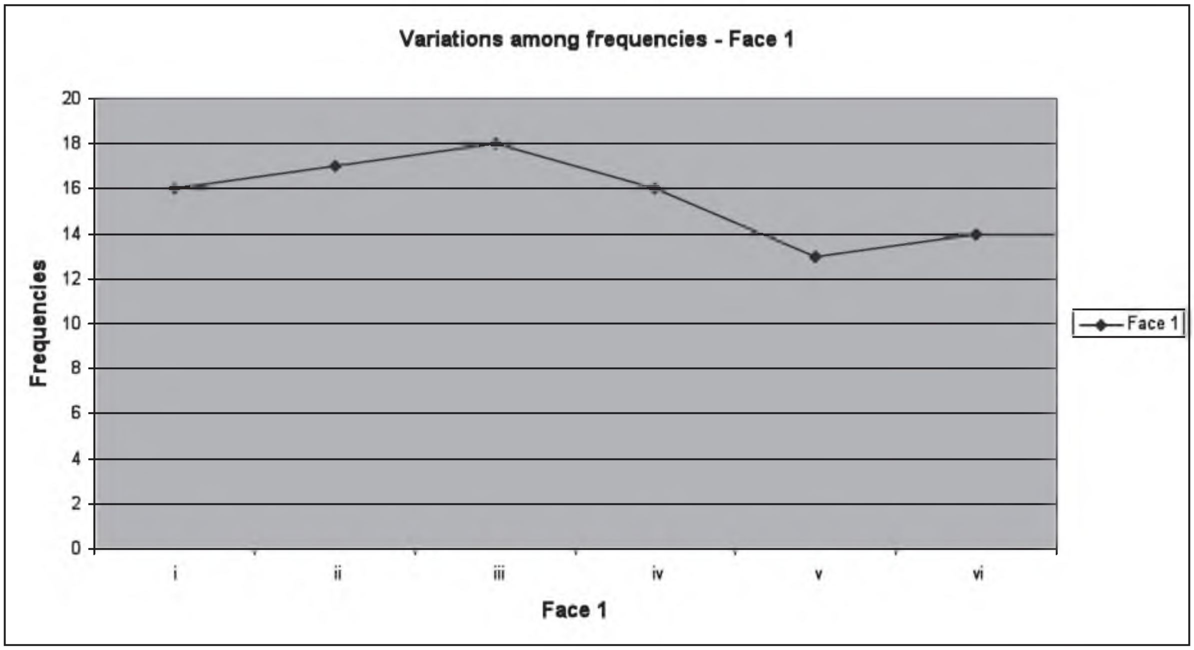
<u>Presentation of Data</u>

The following comparative bar chart illustrates the variations of frequencies for obtaining the numbers 1 through 6 on the uppermost face for experiments (i) and (ii).




Graphs to illustrate experiments (iii), (iv), (v) and (vi) may be shown as well.

The following line graph illustrates the variations among the frequencies for face 1.




Graphs for each of faces 2, 3, 4, 5, and 6 may be shown.

Mathematical Knowledge/Analysis of Data

Choosing to use the different methods for carrying out these experiments, as described in Data Collection, took into account that different conditions of the throws of the die may have significant influences in the outcomes of these throws. The size of the cups chosen may have a particular influence on these outcomes. The inside surface of the two types of cups chosen are also factors that may influence these outcomes. The number of times the die is tossed around in the palm of the hand and/or the number of times it is tossed around in the cups may influence these outcomes. The different coverings of the surface of the table top may also influence these outcomes.

In the absence of more in-depth and elaborate statistical techniques, these simple experiments were intended to give some idea of the theory of classical probability. The limiting relative frequency of an event over a long series of trials is the conceptual foundation of the frequency interpretation of probability. In this framework, it is assumed that as the length of the series increases without bound, the fraction of the experiments in which we observe the event will stabilize.

120 throws under each of the conditions selected should allow for simple comparison of the observed and theoretical frequencies.

Using the principle of relative probability, the following table shows the probability distribution for Experiment (i) and the theoretical probability of obtaining any of the faces numbered 1, 2, 3, 4, 5, 6 landing uppermost.

| # on face | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Relative probability | $\frac{2}{15}$ = 0.13 | $\frac{7}{60}$ = 0.12 | $\frac{9}{40}$ = 0.23 | $\frac{1}{8}$ = 0.13 | $\frac{5}{24}$ = 0.21 | $\frac{23}{120}$ = 0.19 |
| Theoretical probability | $\frac{1}{6}$ = 0.17 | $\frac{1}{6}$ = 0.17 | $\frac{1}{6}$ = 0.17 | $\frac{1}{6}$ = 0.17 | $\frac{1}{6}$ = 0.17 | $\frac{1}{6}$ = 0.17 |

Comparisons of the differences of the observed and theoretical frequencies for 120 throws of the die under the conditions described should be considered as sufficient for an explanation of any significant variation in determining whether the die was biased in favour of any particular face. Barring any significant variation among the relative frequencies, it may be reasoned that the die is not biased.

The relative probabilities can also be calculated for Experiments (ii) through (vi)

Furthermore we can combine the results of all six experiments to arrive at an overall probability for each face as shown in the table below:

| # on face | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Relative frequency | $\frac{96}{720}$ = 0.13 | $\frac{124}{720}$ = 0.17 | $\frac{130}{720}$ = 0.18 | $\frac{128}{720}$ = 0.18 | $\frac{123}{720}$ = 0.17 | $\frac{121}{720}$ = 0.17 |

The above table clearly shows that the relative frequency of each face is close to the true probability (0.17) when the number of trials (720) is large. This is strong evidence to claim that the die is unbiased even though there were differences among the observed frequencies for the six experiments.

Further analysis must be taken in light of any limitations that the project may have. Considering the mean and standard deviation of each of these experiments, account may be taken of the size of the variations of the observed and theoretical values. This aspect may explain any significant variation from the expected mean and variance of these outcomes

The standard deviations for the frequencies of faces 1 through 6 for Experiments (i), (ii), (iii), (iv), (v) and (vi) are 1.67, 1.71, 1.62, 1.63, 1.63 and 1.60 respectively.

Except for Face #2 and to a lesser extent (Face #1), the variances among the outcomes do not appear to suggest significant differences in the results.

## Conclusions

These experiments can be considered simplistic but reasonably effective for the purpose of determining bias in an ordinary gaming die. The number of throws, 120, may be considered sufficient for obtaining relative frequencies and relative probability for the experiments. Increasing the number of throws should result in observed frequencies very close to the theoretical frequencies.

Further statistical analyses can explain variations between the observed and theoretical results. These experiments may be refined by using other methods of throwing the die. Results can be compared for similarity among these results and for a reasonable conclusion about fairness of the die.

**Procedures for Reporting and Submitting School Based Assessment**

(i) Teachers are required to record the mark awarded to each candidate under the appropriate profile dimension on the mark sheet provided by CXC. The completed mark sheets should be submitted to CXC no later than April 30 of the year of the examination.

**Note: The school is advised to keep a copy of the project for each candidate as well as copies of the mark sheets.**

(ii) Teachers will be required to submit to CXC copies of the projects of a sample of candidates as indicated by CXC. This sample will be re-marked by CXC for moderation purposes.

**Moderation of School Based Assessment**

The candidate's performance on the project will be moderated. The standard and range of marks awarded by the teacher will be adjusted where appropriate. However, the rank order assigned by the teacher will be adjusted only in special circumstances and then only after consideration of the data provided by the sample of marked projects submitted by the teacher and re-marked by CXC.

## ♦ RESOURCES

The following is a list of books and other printed material that might be used for Additional Mathematics. The list is by no means exhaustive. Each student should have access to at least one text.


Talbert, J. F. And Heng, H. H. *Additional Mathematics – Pure and Applied*, Singapore: Longman Publishers, 1991.


Website:
http://www.saskschools.ca/curr_content/physics30kindy n/ for kinematics

## ♦ GLOSSARY

**KEY TO ABBREVIATIONS**

K - Knowledge
C - Comprehension
R - Reasoning

| WORD | DEFINITION | NOTES |
|---|---|---|
| analyse | examine in detail | |
| annotate | add a brief note to a label | Simple phrase or a few words only. |
| apply | use knowledge/principles to solve problems | Make inferences/conclusions. |
| assess | present reasons for the importance of particular structures, relationships or processes | Compare the advantages and disadvantages or the merits and demerits of a particular structure, relationship or process. |
| calculate | arrive at the solution to a numerical problem | Steps should be shown; units must be included. |
| classify | divide into groups according to observable characteristics | |
| comment | state opinion or view with supporting reasons | |
| compare | state similarities and differences | An explanation of the significance of each similarity and difference stated may be required for comparisons which are other than structural. |
| construct | use a specific format to make and/or draw a graph, histogram, pie chart or other representation using data or material provided or drawn from practical investigations, build (for example, a model), draw scale diagram | Such representations should normally bear a title, appropriate headings and legend. |

| WORD | DEFINITION | NOTES |
|---|---|---|
| deduce | make a logical connection between two or more pieces of information; use data to arrive at a conclusion | |
| define | state concisely the meaning of a word or term | This should include the defining equation/formula where relevant. |
| demonstrate | show; direct attention to... | |
| derive | to deduce, determine or extract from data by a set of logical steps some relationship, formula or result | This relationship may be general or specific. |
| describe | provide detailed factual information of the appearance or arrangement of a specific structure or a sequence of a specific process | Description may be in words, drawings or diagrams or any appropriate combination. Drawings or diagrams should be annotated to show appropriate detail where necessary. |
| determine | find the value of a physical quantity | |
| design | plan and present with appropriate practical detail | Where hypotheses are stated or when tests are to be conducted, possible outcomes should be clearly stated and/or the way in which data will be analyzed and presented. |
| develop | expand or elaborate an idea or argument with supporting reasons | |
| diagram | simplified representation showing the relationship between components | |
| differentiate/distinguish (between/among) | state or explain briefly those differences between or among items which can be used to define the items or place them into separate categories | |
| discuss | present reasoned argument; consider points both for and against; explain the | |

| WORD | DEFINITION | NOTES |
|---|---|---|
| | relative merits of a case | |
| draw | make a line representation from specimens or apparatus which shows an accurate relation between the parts | In the case of drawings from specimens, the magnification must always be stated. |
| estimate | make an approximate quantitative judgement | |
| evaluate | weigh evidence and make judgements based on given criteria | The use of logical supporting reasons for a particular point of view is more important than the view held; usually both sides of an argument should be considered. |
| explain | give reasons based on recall; account for | |
| find | locate a feature or obtain as from a graph | |
| formulate | devise a hypothesis | |
| identify | name or point out specific components or features | |
| illustrate | show clearly by using appropriate examples or diagrams, sketches | |
| interpret | explain the meaning of | |
| investigate | use simple systematic procedures to observe, record data and draw logical conclusions | |
| justify | explain the correctness of | |
| label | add names to identify structures or parts indicated by pointers | |
| list | itemize without detail | |
| measure | take accurate quantitative readings using appropriate instruments | |

| WORD | DEFINITION | NOTES |
|---|---|---|
| name | give only the name of | No additional information is required. |
| note | write down observations | |
| observe | pay attention to details which characterize a specimen, reaction or change taking place; to examine and note scientifically | Observations may involve all the senses and/or extensions of them but would normally exclude the sense of taste. |
| outline | give basic steps only | |
| plan | prepare to conduct an investigation | |
| predict | use information provided to arrive at a likely conclusion or suggest a possible outcome | |
| record | write an accurate description of the full range of observations made during a given procedure | This includes the values for any variable being investigated; where appropriate, recorded data may be depicted in graphs, histograms or tables. |
| relate | show connections between; explain how one set of facts or data depend on others or are determined by them | |
| sketch | make a simple freehand diagram showing relevant proportions and any important details | |
| state | provide factual information in concise terms outlining explanations | |
| suggest | offer an explanation deduced from information provided or previous knowledge. (... a hypothesis; provide a generalization which offers a likely explanation for a set of data or observations.) | No correct or incorrect solution is presumed but suggestions must be acceptable within the limits of scientific knowledge. |
| use | apply knowledge/principles to solve problems | Make inferences/conclusions. |

***Western Zone Office***
***3 May 2010***

# CARIBBEAN EXAMINATIONS COUNCIL

## Caribbean Advanced Proficiency Examination
## CAPE®

# ADDITIONAL MATHEMATICS

# Specimen Papers and
# Mark Schemes/Keys

| | | |
|---|---|---|
| **Specimen Papers:** | - | Unit 1, Paper 01<br>(Mark Scheme included)<br>Unit 1, Paper 02<br>Unit 1, Paper 03/2 |
| **Mark Schemes and Keys:** | - | Unit 1, Paper 02<br>Unit 1, Paper 03/2 |

# C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

## SECONDARY EDUCATION CERTIFICATE EXAMINATION

## ADDITIONAL MATHEMATICS

## SPECIMEN PAPER

### Paper 01 – General Proficiency

***90 minutes***

**READ THE FOLLOWING INSTRUCTIONS CAREFULLY.**

**1.** This paper consists of **45** items. You will have 90 minutes to answer them.

**2.** In addition to this test booklet, you should have an answer sheet.

**3.** Each item in this test has four suggested answers, lettered (A), (B), (C) and (D). Read each item you are about to answer and decide which choice is best.

**4.** On your answer sheet, find the number which corresponds to your item and shade the space having the same letter as the answer you have chosen. Look at the sample item below.

Sample Item

Evaluate $(4^{-2})^2 \div (1/16)^2$

(A) $4^{-2}$
(B) $4^{-1}$
(C) $4^{0}$
(D) $4^{2}$

Sample Answer

A B ● D

The best answer to this item is "     ", so answer space (C) has been shaded.

**5.** If you want to change your answer, erase it completely before you fill in your new choice.

**6.** When you are told to begin, turn the page and work as quickly and as carefully as you can. If you cannot answer an item, omit it and go on to the one. You can return later to the item omitted. You score will be the number of correct answers produced.

**7.** You may do any rough work in the booklet.

**8.** You may use a silent non-programmable calculator to answer questions.

1. Given that f $(x) = x^3 + 2x^2 - 5x + k$, and that $x - 2$ is a factor of f $(x)$ then $k$ is equal to

(A) -6
(B) -2
(C) 2
(D) 6

2. $a(b + c) - b(a + c)$ is equal to

(A) $a(c - b)$
(B) $a(b - c)$
(C) $c(a - b)$
(D) $c(b - a)$

3. The value of $\sum_{r=1}^{20}(3r - 1)$ is

(A) 590
(B) 610
(C) 650
(D) 1220

4. A teacher illustrates AP's by cutting a length of string into 10 pieces so that the lengths of the pieces are in arithmetic progression and the entire length of the string is used up exactly. If the first piece measures 30 cm and the fourth piece measures 24 cm, the total length of the string is

(A) 60 cm
(B) 210 cm
(C) 240 cm
(D) 390 cm

5. The first term of a GP is 16 and the fifth term is 81. Given that the common ratio is positive, the value of the 4$^{th}$ term is

(A) $\frac{81}{16}$
(B) 24
(C) 54
(D) 64

6. The first four terms of a convergent GP is given by 81, 27, 9, 3. The sum to infinity of this GP is

(A) 54
(B) 120.5
(C) 121.5
(D) 243

7. Given that $2 \times 4^{x+1} = 16^{2x}$, the value of $x$ is

(A) -1
(B) $\frac{1}{4}$
(C) $\frac{1}{3}$
(D) $\frac{1}{2}$

8. $\sqrt[n]{2 \times 4^m}$ is equal to

(A) $\sqrt[n]{8^m}$
(B) $2^{n+2m}$
(C) $2^{n+mn}$
(D) $2^{\frac{2m+1}{n}}$

9. Given that $\log_2 x + \log_2(6x + 1) = 1$, the value of $x$ is

(A) $-\frac{2}{3}$
(B)
(C) $\frac{2}{3}$
(D) $\frac{3}{2}$

**10.** The value of

$\log_4(8) - \log_4(2) + \log_4\left(\frac{1}{16}\right)$ is

(A) -1

(B) $\frac{1}{2}$

(C) 3

(D) 4

**11.** The expression $\frac{1+\sqrt{3}}{\sqrt{3}-1}$ when simplified is equal to

(A) -1

(B) 1

(C) $\frac{\sqrt{3}+2}{2}$

(D) $\sqrt{3}+2$

**12.** $f(x) = -5 - 8x - 2x^2$. By completing the square f ($x$) can be expressed as

(A) $2(x+2)^2 - 4$

(B) $4 - 2(x-2)^2$

(C) $3 - 2(x+2)^2$

(D) $3 - 2(x-2)^2$

**13.** The roots of the equation $2x^2 - x + 1 = 0$ are

(A) real and equal
(B) real and distinct
(C) not real and equal
(D) not real and distinct

**14.** For $-2 \le x \le 2$, the maximum value of $4-(x+1)^2$, and the value of $x$ for which $4-(x+1)^2$ is maximum are respectively

(A) 5 and 1
(B) 2 and -1
(C) 4 and -1
(D) 4 and 1

**15.** $f(x) = x(x+5)+6$. Given that f ($x$) is one-to-one for $x \ge k$, the value of $k$ is

(A) $-\frac{5}{2}$

(B) $-\frac{2}{5}$

(C) $\frac{2}{5}$

(D) $\frac{5}{2}$

**16.** If a function f is defined by $f : x \rightarrow \frac{x+3}{x-1}$, $x \ne 1$, then $f^{-1}(-4)$ is equal to

(A) -1

(B) $\frac{1}{5}$

(C) 1

(D) 5

**17.** A function g is defined by $g : x \rightarrow 3x - 1$. Expressed in terms of $a$, g ($3a$ - 1) is

(A) $9a$ - 1
(B) $3a$ - 4
(C) $9a$ - 2
(D) $9a$ - 4

**18.** Functions f and g are defined by
$f : x \to 3x - 2$ and
$g : x \to \frac{12}{x} - 4, \ x \neq 0$.

The composite function fg is defined by

(A) $fg : x \to \frac{36}{x} - 4, \ x \neq 0$

(B) $fg : x \to \frac{12}{x} - 12, \ x \neq 0$

(C) $fg : x \to \frac{12}{x} - 6, \ x \neq 0$

(D) $fg : x \to \frac{36}{x} - 14, \ x \neq 0$

**19.** The range of values for which $2x^2 < 5x + 3$ is

(A) $-\frac{1}{2} < x < 3$

(B) $\frac{1}{2} < x < 3$

(C) $x < -\frac{1}{2}$ and $x < 3$

(D) $x > -\frac{1}{2}$ and $x > 3$

**20.** The values of $x$ which satisfy the inequality $\frac{2x - 3}{x + 1} > 0$ are

(A) $x > -1$ and $x > \frac{3}{2}$

(B) $x > \frac{3}{2}$

(C) $x < -1$ or $x > \frac{3}{2}$

(D) $x > -1$

**21.** The coordinates of the points $A$ and $B$ are (2, -3) and (-10, -5) respectively. The perpendicular bisector to the line $AB$ is given by the equation

(A) $x - 6y + 20 = 0$

(B) $6x + y + 28 = 0$

(C) $x + 6y - 20 = 0$

(D) $6x + y - 28 = 0$

**22.** The lines $2y - 3x - 13 = 0$ and $y + x + 1 = 0$ intersect at the point $P$, where the coordinates of P are

(A) (3, 2)
(B) (3, -2)
(C) (-3, -2)
(D) (-3, 2)

**23.** The radius, $r$, and the coordinates of the centre, $C$, of the circle with equation $x^2 + y^2 - 6x + 4y - 12 = 0$ are

(A) $r = 5$, $C$ (-2, 3),

(B) $r = 25$, $C$ (2, -3),

(C) $r = 12$, $C$ (-3, 2),

(D) $r = 5$, $C$ (3, -2),

**24.** If the length of the vector $\mathbf{p} = 2\mathbf{i} - k\mathbf{j}$ is $\sqrt{13}$ and $k$ is real, then

I. $k = 3$
II. $k = -3$
III. $k = \sqrt{17}$
IV. $k = -\sqrt{17}$

(A) I or II only
(B) I or III only
(C) I or IV only
(D) II or IV only

**25**. The value of the real number $t$ for which the two vectors $\mathbf{a} = 4\mathbf{i} + t\mathbf{j}$ and $\mathbf{b} = 2\mathbf{i} - 3\mathbf{j}$ are parallel is

(A) -6

(B) $-\frac{3}{4}$

(C) $\frac{4}{3}$

(D) 6

**26**. The position vectors of $A$ and $B$ relative to an origin O are $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 7 \\ 4 \end{pmatrix}$ respectively. The acute angle $AOB$ is given by

(A) $\cos^{-1}\left(\frac{2}{\sqrt{65}}\right)$

(B) $\cos^{-1}\left(\frac{\sqrt{26}}{13 \times 65}\right)$

(C) $\cos^{-1}\left(\frac{\sqrt{2}}{\sqrt{65}}\right)$

(D) $\cos^{-1}\left(\frac{26}{\sqrt{13}\,\sqrt{65}}\right)$

**27**. The trigonometrical expression $\frac{1 + \sin x}{\cos x} + \frac{\cos x}{1 + \sin x}$ is identical to

(A) 1

(B) $\frac{2}{\cos x}$

(C) $\frac{1 + \sin x + \cos x}{\cos x(1 + \sin x)}$

(D) $\frac{2}{\cos x(1 + \sin x)}$

**28**. $\cos(A - B) - \cos(A + B) \equiv$

(A) $2 \sin A \sin B$
(B) $-2 \sin A \cos B$
(C) $2 \cos A \sin B$
(D) $2 \cos A \cos B$

**29**. If $\sin \theta = \frac{15}{17}$ and $\theta$ is obtuse, then $\cos \theta$ is equal to

(A) $-\frac{8}{15}$

(B) $-\frac{8}{17}$

B. $\frac{8}{15}$

(D) $\frac{8}{17}$

**30**. The smallest positive angle for which the equation $\sin \theta + \cos \theta = 0$ is

(A) $\frac{\pi}{4}$

(B) $\frac{3\pi}{4}$

(C) $\frac{5\pi}{4}$

(D) $\frac{7\pi}{4}$

**31**. For $0 \le \theta \le 2\pi$, solutions for the equation $4 \sin^2 \theta - 1 = 0$ exist in quadrants

(A) 1, 2 and 3
(B) 1, 3 and 4
(C) 2, 3 and 4
(D) 1, 2, 3 and 4

32. $2\sin\left(x - \frac{\pi}{2}\right)$ is equal to

(A) $2\sin x - 2$

(B) $-2\cos x$

(C) $2\cos\left(x + \frac{\pi}{2}\right)$

(D) $2\sin x - \pi$

33. For which of the following ranges of values is f ($x$) = 2 + cos 3$x$ valid?

(A) $1 \le f(x) \le 3$
(B) $-1 \le f(x) \le 1$
(C) $-2 \le f(x) \le 2$
(D) $0 \le f(x) \le 2$

34. For $0 \le x \le 2\pi$, the values of $x$ which satisfy the equation $2\cos^2 x + 3\sin x = 0$ are

(A) $x = \frac{\pi}{6}, x = \frac{5\pi}{6}$

(B) $x = -\frac{\pi}{6}, x = -\frac{5\pi}{6}$

(C) $x = \frac{7\pi}{6}, x = \frac{11\pi}{6}$

(D) $x = \frac{5\pi}{6}, x = \frac{7\pi}{6}$

35. Given that $y = (3x - 2)^3$, then $\frac{dy}{dx} =$

(A) $3(3x - 2)^2$

(B) $3(3x)^2$

(C) $3(3x - 2)^3$

(D) $9(3x - 2)^2$

36. Given that $y = \frac{3x+5}{2x-11}$, then $\frac{dy}{dx} =$

(A) $\frac{(3x+5)(2) + (2x-11)(3)}{(2x-11)^2}$

(B) $\frac{(2x-11)(3) + (3x+5)(2)}{(2x-11)^2}$

(C) $\frac{(2x-11)(3) - (3x+5)(2)}{(2x-11)^2}$

(D) $\frac{(3x+5)(2) - (2x-11)(3)}{(2x-11)^2}$

37. The curve $C$ is given by the equation $y = 3\sin x + 2$. The value of $\frac{dy}{dx}$ at the point where $x = \frac{\pi}{3}$ is

(A) $\frac{1}{2}$

(B) $\frac{3}{2}$

(C) $\frac{7}{2}$

(C) 3

38. The point $P$ (2, 2) lies on the curve with equation $y = x(x-3)^2$. The equation of the normal to the curve at the point $P$ is given by

(A) $y - 2 = 3(x - 2)$

(B) $y - 2 = -3(x - 2)$

(C) $y - 2 = \frac{1}{3}(x - 2)$

(D) $y - 2 = \frac{-1}{3}(x - 2)$

**39**. The curve $C$ is given by the equation $y = 4x + \frac{9}{x}$. The second derivative, $\frac{d^2y}{dx^2}$, is given by

(A) $4 + \frac{9}{x^3}$

(B) $\frac{18}{x^3}$

C. $4 - \frac{9}{x^3}$

(D) $-\frac{9}{2x^3}$

**40**. The positive value of $z$ for which $\int_0^z x^2 \, dx = 9$ is

(A) 3
(B) 4.5
(C) 9
(D) 27

**41**. The gradient of the tangent to a curve $C$ at $(x, y)$ is given by $\frac{dy}{dx} = \frac{1}{(3x+4)^2}$. The curve passes through the point $P\left(-1, \frac{2}{3}\right)$. The equation of the curve $C$ is given by

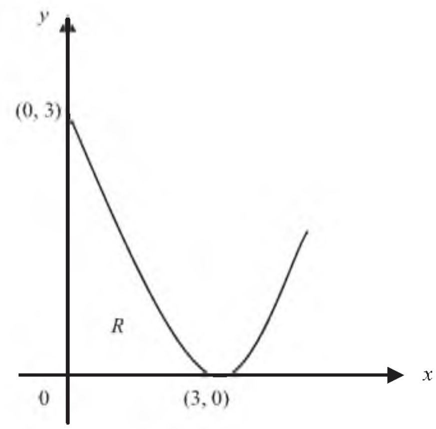
(A) $y = \frac{2}{(3x+4)} + 1$

(B) $y = \frac{-6}{(3x+4)^3}$

(C) $y = \frac{-2}{3(3x+4)} + 4$

(D) $y = \frac{-1}{3(3x+4)} + 1$

<u>Item **42**</u> refers to the figure below.




**42**. The finite region $R$ is bounded by the $y$-axis, the $x$-axis, and the arc of the curve $y = (x-3)^2$ as shown in the figure above. The area of $R$ in square units is

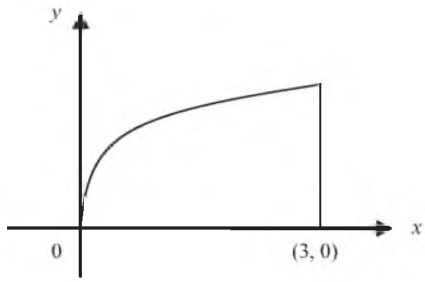
(A) 1

(B) 3

(C) 9

(D) 27

**Item 43** refers to the figure below.




**43.** The finite region enclosed by the curve $y = \sqrt{x}$, $x \geq 0$, the $x$-axis and the line $x = 3$, as shown in the figure above, is rotated completely about the $x$-axis. The volume of the solid of revolution formed is given by

(A) $\int_0^3 \sqrt{x}\, dx$

(B) $\pi \int_0^3 x\, dx$

(C) $\pi \int_0^3 \sqrt{x}\, dx$

(D) $\pi \int_0^3 x^2\, dx$

**44.** $\int (2x + 3)^5\, dx =$

(A) $\left[\frac{1}{6}(2x + 3)^6\right] + C$

(B) $\left[\frac{1}{2}(2x + 3)^6\right] + C$

(C) $\left[\frac{1}{12}(2x + 3)^6\right] + C$

(D) $\left[\frac{1}{3}(2x + 3)^6\right] + C$

**45.** Given $\frac{dy}{dx} = 3 \sin x - 2 \cos x$, the indefinite integral is given by

(A) $y = 3 \cos x - 2 \sin x + C$

(B) $y = -3 \cos x + 2 \sin x + C$

(C) $y = -3 \cos x - 2 \sin x + C$

(D) $y = 3 \cos x + 2 \sin x + C$

**END OF TEST**

TEST CODE 01254020/SPEC

**FORM TP 2011037/SPEC**

**C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L**

**SECONDARY EDUCATION CERTIFICATE EXAMINATION**

**ADDITIONAL MATHEMATICS**

**SPECIMEN PAPER**

**Paper 02 – General Proficiency**

***2 hours and 40 minutes***

**INSTRUCTIONS TO CANDIDATES**

**1.** DO NOT open this examination paper until instructed to do so.

**2.** This paper consists of FOUR sections. Answer ALL questions in Section 1, Section 2 and Section 3.

**3.** Answer ONE question in Section 4.

**4.** Write your solutions with full working in the booklet provided.

**Required Examination Materials**

Electronic calculator (non programmable)
Geometry Set
Mathematical Tables (provided)
Graph paper (provided)

## SECTION 1

**Answer BOTH questions.**

**All working must be clearly shown.**

**1.** (a) A function $f: R \rightarrow R$ is defined on domain R and co-domain R, where R is the set of real numbers, by the rule

$$f(x) = x^2$$

(i) State with reason, whether $f$ is many-to-one or one-to-one.

**[1 mark]**

(ii) If instead, the domain of $f$ is the set of non-negative real numbers,

a) Determine a function $g$ such that $g[f(x)] = x$ for all values of $x$ in this domain. **[1 mark]**

b) On the same pair of axes, sketch the graphs of $f$ and $g$. **[2 mark]**

(b) Use the remainder theorem, or otherwise, to find the reminder when $x^3 - 2x^2 + 4x - 21$ is divided by $x - 3$.

**[2 marks]**

(c) A student collects laboratory data for two quantities $q$ and $p$ as shown in Table 1.

**Table 1**

| $q$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $p$ | 0.50 | 0.63 | 0.72 | 0.80 |

The student reasons a relationship of the form $p = aq^n$

(i) Use logarithms to reduce this relation to a linear form.

**[2 marks]**

(ii) Using the graph paper provided and a scale of 1 cm to represent 0.1 units on the horizontal axis and a scale of 2 cm to represent 0.1 units on the vertical axis, plot a suitable straight line graph and hence estimate the constants $a$ and $n$.

**[6 marks]**

**Total 14 marks**

**2.** (a) Let $f(x) = 3x^2 + 12x - 18$.

(i) Express $f(x)$ in the form $a(x + b)^2 + c$. **[3 marks]**

(ii) State the minimum value of $f(x)$. **[1 mark]**

(iii) Determine the value of $x$ for which $f(x)$ is a minimum. **[1 mark]**

(b) Find the set of values of $x$ for which $2x^2 + 2 > 5x$. **[4 marks]**

(c) Given the series $\frac{1}{2} + \frac{1}{2^4} + \frac{1}{2^7} + \frac{1}{2^{10}} + \ldots$

(i) show that this series is geometric, **[3 marks]**

(ii) hence, find the sum to infinity of this series. **[2 marks]**

**Total 14 marks**

## SECTION 2

**Answer BOTH Questions**

3. (i) Write the equation of the circle $C$, with centre $(-1, 2)$ and radius $\sqrt{13}$ units.

**[1 mark]**

(ii) Find the equation of the tangent to the circle $C$ at the point P(2,4).

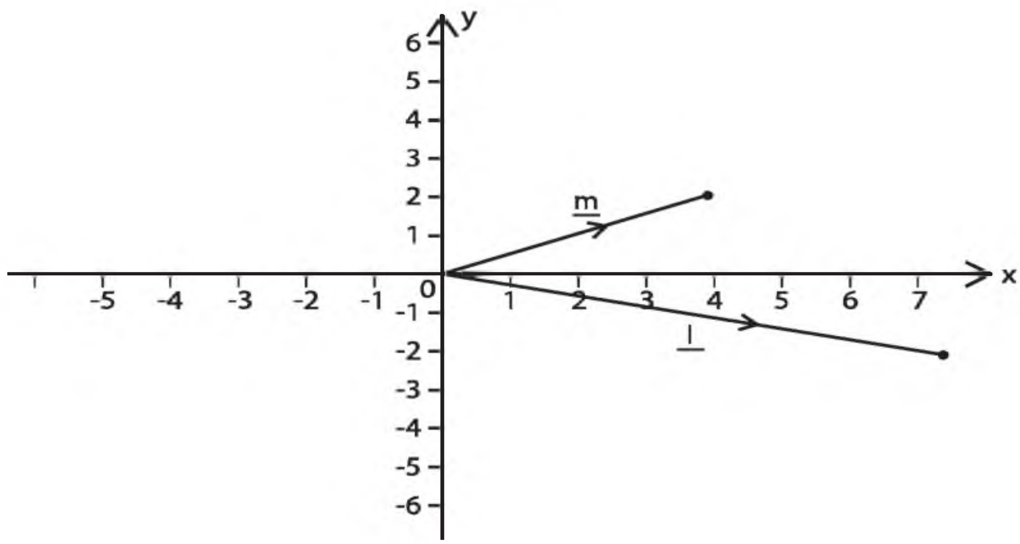
**[4 marks]**

(b) The position vector of two points, A and B, relative to a fixed origin O, are $3t\mathbf{i} + 2t\mathbf{j}$ and $4\mathbf{i} - 2t\mathbf{j}$ respectively, where $t > 0$.

Find the value of $t$ such that $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are perpendicular.

**[4 marks]**

(c) The points L and M referred to a fixed origin O are represented by the vectors $\mathbf{l} = 7\mathbf{i} - 2\mathbf{j}$ and $\mathbf{m} = 4t\mathbf{i} + 2\mathbf{j}$ respectively, as shown in the diagram below.

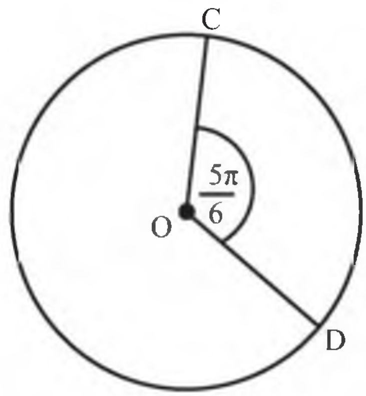



Find the unit vector in the direction of $\overrightarrow{LM}$.

**[3 marks]**

**Total 12 marks**

**4.** (a) The diagram below shows a circle of centre O and radius 6 cm. The sector COD subtends the angle $\frac{5\pi}{6}$ at the centre.




Working in **radians**, calculate, giving your answers in terms of π,

(i) the length of the minor arc CD

**[1 mark]**

(ii) the area of the minor sector OCD

**[2 marks]**

(b) (i) Given that $\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$, where $x$ is acute, show that

$$\sin\left(x - \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}(\sin x - \cos x).$$

**[2 marks]**

(ii) Using the fact that $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ and $\cos\frac{\pi}{3} = \frac{1}{2}$, find the exact value of $\sin\frac{\pi}{12}$ showing <u>ALL</u> steps in your working.

**[3 marks]**

(c) Prove the identity $\left(\tan\theta - \frac{1}{\cos\theta}\right)^2 \equiv -\frac{\sin\theta - 1}{\sin\theta + 1}$.

**[4 marks]**

**Total 12 marks**

## SECTION 3

**Answer BOTH Questions**

**5.** (a) Differentiate the following with respect to $x$, simplifying your result as far as possible

$$(5 - 2x)(1 + x)^4$$

**[4 marks]**

(b) The point P lies on the curve $y = x^2$. The value of $x$ at P is -2.

Find the equation of the tangent to the curve at P.

**[4 marks]**

(c) Find the stationary points on the curve $f(x) = 2x^3 - 9x^2 + 12x$ and distinguish their nature.

**[6 marks]**

**Total 14 marks**

**6.** (a) Evaluate $\int_1^2 (3x - 1)^2 \, dx$. **[4 marks]**

(b) Evaluate $\int_0^{\frac{\pi}{2}} (5 \sin x - 3 \cos x) \, dx$. **[4 marks]**

(c) A curve passes through the point P $\left(0, \frac{7}{2}\right)$ and is such that $\frac{dy}{dx} = 2 - x$.

(i) Find the equation of the curve. **[3 marks]**

(ii) Find the area of the finite region bounded by the curve, the $x$-axis, the $y$-axis and the line $x=5$.

**[3 marks]**

**Total 14 marks**

## SECTION 4

Answer Only **ONE** Question

**7.** (a) In a Lower Sixth Form there are 43 students who are studying either Statistics or Physics or both Statistics and Physics. 28 students study Statistics and 19 students study Physics. If a student is selected at random, what is the probability that he/she is studying

(i) both Statistics and Physics, **[3 marks]**

(ii) Physics only. **[2 marks]**

(b) A tetrahedral die has four faces. Two fair tetrahedral dice, one red and one blue, have faces numbered 0, 1, 2, and 3 respectively. The dice are rolled and the numbers face down on the two dice are recorded. $R$ is the score on the red die and $B$ is the score on the blue die.

(i) Find $P(R = 3 \text{ and } B = 0)$.

**[2 marks]**

The random variable $T$ is $R$ multiplied by $B$.

(ii) Complete the diagram below to represent the sample space that shows all the possible values of $T$

| | | | | |
|---|---|---|---|---|
| **3** | | | | |
| **2** | | 2 | | |
| **1** | 0 | | | |
| **0** | | | | |
| B \ R | **0** | **1** | **2** | **3** |

**Sample space diagram of T**

**[3 marks]**

The table below gives the probability of each possible value of $t$.

| $t$ | 0 | 1 | 2 | 3 | 4 | 6 | 9 |
|---|---|---|---|---|---|---|---|
| $P(T = t)$ | $a$ | $\frac{1}{16}$ | $\frac{1}{8}$ | $b$ | $c$ | | |

(iii) Find the values of a, $b$ and $c$.

**[3 marks]**

**7.** (c) The number of cars parked on a local beachfront on each night in August last year is summarized in the following stem and leaf diagram.

| | |
|---|---|
| 1 | 0 5 |
| 2 | 1 2 4 8 |
| 3 | 0 3 3 3 4 7 8 8 |
| 4 | 1 1 3 5 8 8 8 9 9 |
| 5 | 2 3 6 6 7 |
| 6 | 2 3 4 |

Key: 1|0 means 10

(i) Find the median and quartiles for these data.

**[3 marks]**

(ii) Construct a box-and-whisker plot to illustrate these data and comment on the shape of the distribution.

**[4 marks]**

**Total 20 marks**

8. (a) A car moves along a horizontal straight road, passing two points *A* and *B*. The speed of the car at *A* is 15m $s^{-1}$. When the driver passes *A*, he sees a warning sign *W* ahead of him, 120 m away. He immediately applies the brakes and the car decelerates uniformly, reaching *W* at a speed of 5 $ms^{-1}$. At *W*, the driver sees that the road is clear. He then immediately accelerates the car with uniform acceleration for 16 secs to reach a speed of $V$ $ms^{-1}$ where $V > 15$. He then maintains a constant speed of $V$ $ms^{-1}$ for 22 secs, passing *B*.

(i) Sketch, on the graph paper provided a velocity-time graph to illustrate the motion of the car as it moves from *A* to *B*.

**[3 marks]**

(ii) Find the time taken for the car to move from *A* to *B*.

**[3 marks]**

The distance from *A* to *B* is 1 km.

(iii) Find the value of $V$.

**[5 marks]**

(b) A particle moves in a straight line so that $t$ seconds after passing through a fixed point $O$, its velocity, $v$ m/s is given by

$$v = 3t^2 - 30t + 72.$$

Calculate the:

(i) values of $t$ when the particle is at instantaneous rest,

**[3 marks]**

(ii) distance moved by the particle during the interval between the two values of $t$ found in b (i).

**[3 marks]**

(iii) total distance moved by the particle between $t = 0$ and $t = 7$.

**[3 marks]**

**Total 20 marks**

**END OF TEST**

TEST CODE 01254030/SPEC

FORM TP 2011038/SPEC

# C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

**HEADQUARTERS**

**SECONDARY EDUCATION CERTIFICATE**
**EXAMINATION**

**ADDITIONAL MATHEMATICS**

**PAPER 03/2**

**ALTERNATIVE**

*90 minutes*

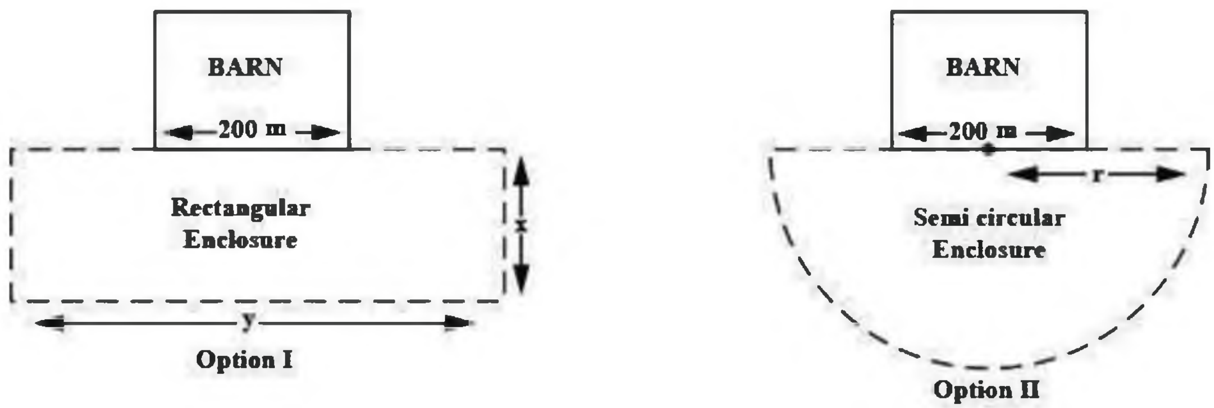
**Answer the given questions**

**DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO**

A farmer plans to construct an enclosure for his goats making use of one side of a barn that is 200 m in length. He has 800 m of fencing material available and wants to maximize the enclosed area.

The farmer must decide whether to make a rectangular or semicircular enclosure as shown in the diagrams below.

You are given that the radius of the semi circular enclosure is $r$, the length of the rectangular enclosure is $x$ and the width is $y$. .




(i) Formulate the given real world problem **mathematically.**

(7 marks)

(ii) Show that for Option I a square enclosure maximizes the area and determine the MAXIMUM possible area.

(7 marks)

(iii) Determine the MAXIMUM area of the enclosure in Option II.

(4 marks)

(iv) Make a recommendation to the farmer on the MOST appropriate enclosure, giving a reason.

(2 marks)

**Total 20 marks**

## CSEC ADDITIONAL MATHEMATICS SPECIMEN PAPER 01

| Item | Key | Specific Objective |
|---|---|---|
| 1 | A | 1A4 |
| 2 | C | 1A1 |
| 3 | B | 1F4 |
| 4 | B | 1F9 |
| 5 | C | 1F7 |
| 6 | C | 1F11 |
| 7 | D | 1E3 |
| 8 | D | 1E2 |
| 9 | B | 1E6 |
| 10 | A | 1E5 |
| 11 | D | 1E1 |
| 12 | C | 1B1 |
| 13 | D | 1B4 |
| 14 | C | 1B2 |
| 15 | A | 1D3 |
| 16 | B | 1D4 |
| 17 | D | 1D7 |
| 18 | D | 1D7 |
| 19 | A | 1C1 |
| 20 | C | 1C2 |
| 21 | B | 2A2 |
| 22 | D | 2A3 |
| 23 | D | 2A5 |
| 24 | A | 2B7 |
| 25 | A | 2B10 |
| 26 | D | 2B9 |
| 27 | B | 2C10 |
| 28 | A | 2C10 |
| 29 | B | 2C4 |
| 30 | B | 2C11 |
| 31 | D | 2C11 |
| 32 | B | 2C8 |
| 33 | A | 2C6 |
| 34 | C | 2C11 |
| 35 | D | 3A8 |
| 36 | C | 3A8 |
| 37 | B | 3A5 |
| 38 | C | 3A17 |
| 39 | B | 3A14 |
| 40 | A | 3B8 |
| 41 | D | 3B9 |
| 42 | C | 3B10(i) |
| 43 | B | 3B10(ii) |
| 44 | C | 3B5 |
| 45 | C | 3B7 |

# C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L
HEADQUARTERS

**SECONDARY EDUCATION CERTIFICATE EXAMINATION**

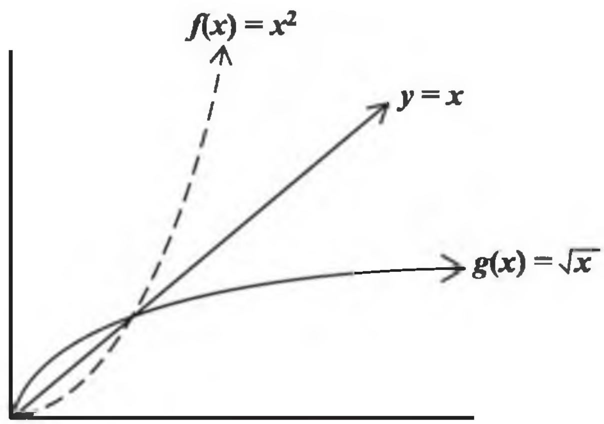
**ADDITIONAL MATHEMATICS**

**PAPER 02**

**SPECIMEN PAPER**
**KEY AND MARK SCHEME**

**ADDITIONAL MATHEMATICS**
**PAPER 02**
**KEY AND MARK SCHEME**

**Question 1**

| | CK | AK | R |
|---|---|---|---|
| (a) (i) $f$ cannot be 1-1 since for example $f(-2) = 4$ and $f(2) = 4$. Two objects can have the same images. Thus $f$ is many-to-one (accept any reasonable explanation or horizontal line test) | | | 1 |
| (ii) a) If $g[f(x)] = x$ then $g$ must be the inverse of $f$. Let $y = x^2$ where $x$ is a non-negative real number. | | 1 | |
| Then $x = \sqrt{y} \Rightarrow g(x) = \sqrt{x}$ | | | |
| or $= \sqrt{(x^2)} = x$, for all $x \geq 0$. | | | |
| b)   | 1 | 1 | |
| (b) Using f (3) = R | | | 1 |
| $f(3) = 3^3 - 2(3)^2 + 4(3) - 21$ | | 1 | |
| $f(3) = 0$ | | | |
| (c) (i) $p = aq^n \Rightarrow \log p = \log a + n \log q$ | 1 | | 1 |

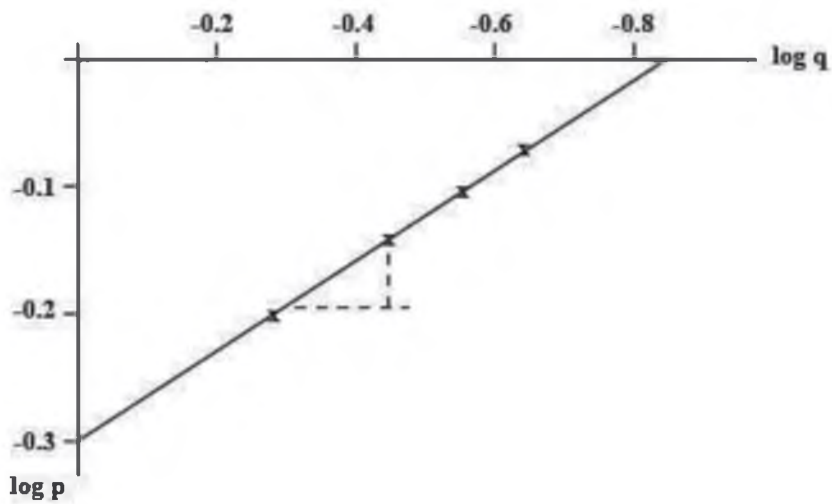
**ADDITIONAL MATHEMATICS**
**PAPER 02**
**KEY AND MARK SCHEME**

(ii)

Plot log $p$ against log $q$: First find the table of values

| log $q$ | 0 | 0.3 | 0.48 | 0.6 |
|---|---|---|---|---|
| log $p$ | -0.30 | -0.20 | -0.14 | -0.10 |

On the graph paper provided plot the points and draw a straight line through the points




The $y$ intercept $c = -0.3 = \log a$

$\therefore\ a = 10^{-0.3} = 0.5$

The slope of the line is $\dfrac{y_2 - y_1}{x_2 - x_1} = m = 0.33$ (by measurement)

$\therefore\ n = 0.33$

(graphical measurements of slope may vary slightly above or below this value)

(Specific Objectives-Sect 1: A3, D3-5, D7, E7-8)

| CK | AK | R |
|---|---|---|
| | 1 | |
| 1 | | |
| | | 1 |
| | 1 | |
| | 1 | |
| | | 1 |
| **3** | **6** | **5** |

**ADDITIONAL MATHEMATICS**
**PAPER 02**
**KEY AND MARK SCHEME**

**Question 2**

| | | | CK | AK | R |
|---|---|---|---|---|---|
| (a) | (i) | $3x^2 + 12x - 18$ | | | |
| | | $3\,[x^2 + 4x] - 18$ | 1 | | |
| | | $3[(x + 2)^2 - 4] - 18$ | | 1 | |
| | | $3(x + 2)^2 - 12 - 18$ | | 1 | |
| | | $3\,(x + 2)^2 - 30$ | | | |
| | (ii) | Minimum value is $y = -30$ | | | 1 |
| | (iii) | Value of $x$ at minimum point is $x = -2$ | | | 1 |
| (b) | | $2x^2 + 2 > 5x$ | | | |
| | | $2x^2 - 5x + 2 > 0$ | | 1 | |
| | | $(2x - 1)(x - 2) > 0$ | | 1 | |
| | | $x < \frac{1}{2}$ | | | 1 |
| | | or $x > 2$ | | | 1 |
| (c) | (i) | Given the series $\frac{1}{2} + \frac{1}{2^4} + \frac{1}{2^7} + \frac{1}{2^{10}} + \ldots$ | | | |
| | | S is a geometric series if $\frac{T_2}{T_1} = \frac{T_n}{T_{n-1}} = r$ | 1 | | |
| | | i.e. S has a common ratio $r$. | | | |
| | | $r = \frac{\frac{1}{2^7}}{\frac{1}{2^4}} = \frac{\frac{1}{2^4}}{\frac{1}{2}}$ | | | 1 |
| | | $r = \frac{1}{2^3} = \frac{1}{8}$ | | 1 | |

**ADDITIONAL MATHEMATICS**
**PAPER 02**
**KEY AND MARK SCHEME**

**Question 2**

**(cont'd)**

(c) (ii)

$$S_\infty = \frac{a}{1-r} = \frac{\frac{1}{2}}{1-\frac{1}{8}}$$

$$S_\infty = \frac{\frac{1}{2}}{\frac{7}{8}} = \frac{4}{7}$$

(Specific Objectives - Sect 1: B1, B2, C1, F6, F11)

| CK | AK | R |
|---|---|---|
| 1 | | |
| | 1 | |
| **3** | **6** | **5** |

**ADDITIONAL MATHEMATICS**
**PAPER 02**
**KEY AND MARK SCHEME**

## Question 3

| | | | CK | AK | R |
|---|---|---|---|---|---|
| (a) | (i) | $(x+1)^2 + (y-2)^2 = (\sqrt{13})^2$ | | | |
| | | $(x+1)^2 + (y-2)^2 = 13$ | 1 | | |
| | (ii) | The gradient, $m$, of the radius through P (2, 4) is $m = \frac{4-2}{2+1}$ | 1 | | |
| | | $= \frac{2}{3}$ | | | |
| | | The gradient of the tangent to circle P is $-\frac{3}{2}$ | | | 1 |
| | | The equation of the tangent at P is given by $\frac{y-4}{x-2} = -\frac{3}{2}$ | | 1 | |
| | | $2y - 8 = -3x + 6$ | | 1 | |
| | | $2y + 3x = 14$ | | | |
| (b) | | $(3t\mathbf{i} + 2t\mathbf{j}) \cdot (4\mathbf{i} - 2t\mathbf{j}) = 0$ | | | 1 |
| | | $12t - 4t^2 = 0$ | | 1 | |
| | | $4t(3 - t) = 0$ | | 1 | |
| | | $t = 0$ or $t = 3$; $t = 3$, since $t > 0$ | | | 1 |
| | | | | | |

**ADDITIONAL MATHEMATICS**
**PAPER 02**
**KEY AND MARK SCHEME**

**Question 3**
**(cont'd)**

| | | CK | AK | R |
|---|---|---|---|---|
| (c) | $\overrightarrow{LM} = \overrightarrow{LO} + \overrightarrow{OM}$ | | | |
| | $= -\begin{pmatrix} 7 \\ -2 \end{pmatrix} + \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ | | | |
| | $= \begin{pmatrix} -3 \\ 4 \end{pmatrix}$ | 1 | | |
| | $= -3\mathbf{i} + 4\mathbf{j}$ | | | |
| | The unit vector in the direction of $\overrightarrow{LM}$ is | | | |
| | $\frac{1}{\sqrt{(-3)^2 + (4)^2}}(-3\mathbf{i} + 4\mathbf{j})$ | 1 | | |
| | $= \frac{1}{5}(-3\mathbf{i} + 4\mathbf{j})$ | | 1 | |
| | (Specific Objectives-Sect 2: A4, A6, B5, B10) | **4** | **5** | **3** |

**ADDITIONAL MATHEMATICS**
**PAPER 02**
**KEY AND MARK SCHEME**

## Question 4

| Working | CK | AK | R |
|---|---|---|---|
| (a) (i) $l_{CD} = r\theta$ | | | |
| $= 6 \times \frac{5\pi}{6} = 5\pi$ cm | | 1 | |
| (ii) $\text{Area}_{\text{sector COD}} = \frac{1}{2} r^2 \theta$ | 1 | | |
| $= \frac{1}{2}(6)^2 \left(\frac{5\pi}{6}\right) = 15\pi$ cm$^2$ | | | 1 |
| (b) (i) $\sin\left(x - \frac{\pi}{4}\right) = \sin x \cos\frac{\pi}{4} - \cos x \sin\frac{\pi}{4}$ | | | 1 |
| $= \frac{\sqrt{2}}{2}\sin x - \frac{\sqrt{2}}{2}\cos x$ | | | |
| $= \frac{\sqrt{2}}{2}(\sin x - \cos x)$ | | 1 | |
| (ii) $\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$ | 1 | | 1 |
| $= \frac{\sqrt{2}}{2}\left(\sin\frac{\pi}{3} - \cos\frac{\pi}{3}\right)$ | | | |
| $= \frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)$ | | 1 | |
| $= \frac{\sqrt{2}}{4}\left(\sqrt{3} - 1\right)$ | | | |

**ADDITIONAL MATHEMATICS**
**PAPER 02**
**KEY AND MARK SCHEME**

**Question 4**
**(cont'd)**

| | | CK | AK | R |
|---|---|---|---|---|
| 4 (c) | $\text{LHS} \equiv \left(\frac{\sin\theta}{\cos\theta} - \frac{1}{\cos\theta}\right)^2$ | | | 1 |
| | $\equiv \left(\frac{\sin\theta - 1}{\cos\theta}\right)^2$ | | | |
| | $\equiv \frac{(\sin\theta - 1)^2}{1 - \sin^2\theta}$ | | 1 | |
| | $\equiv \frac{(\sin\theta - 1)^2}{(1 - \sin\theta)(1 + \sin\theta)}$ | | 1 | |
| | $\equiv -\frac{\sin\theta - 1}{\sin\theta + 1}$ | | | 1 |
| | (Specific objectives-Sect 2: C3, C5, C8, C10) | **2** | **5** | **5** |

**ADDITIONAL MATHEMATICS**
**PAPER 02**
**KEY AND MARK SCHEME**

## Question 5

| | | CK | AK | R |
|---|---|---|---|---|
| (a) | $\frac{d}{dx}(5-2x)(1+x)^4 = (1+x)^4\frac{d}{dx}(5-2x) + (5-2x)\frac{d}{dx}(1+x)^4$ | 1 | | |
| | $= (1+x)^4(-2) + (5-2x)\left[4(1+x)^3\ (1)\right]$ | | 1 | |
| | $= 2(1+x)^3(-1-x+10-4x)$ | | | 1 |
| | $= 2(1+x)^3(9-5x)$ | | 1 | |
| (b) | $\frac{dy}{dx} = 2x$ | 1 | | |
| | $\left(\frac{dy}{dx}\right)_{(-2,\,4)} = -4$ | | | 1 |
| | $\text{tangent}_p: \quad y-4=-4(x+2)$ | | 1 | |
| | $4x+y+4=0$ | | 1 | |
| (c) | $\frac{d}{dx}\left(2x^3-9x^2+12x\right) = 6x^2-18x+12$ | | 1 | |
| | $6x^2-18x+12=0 \Rightarrow (x-1)(x-2)=0$ | | | 1 |
| | $x=1, y=5 \quad x=2, y=4$ | | 1 | |
| | $\frac{d^2y}{dx^2} = 12x-18$ | | | 1 |
| | $\left(\frac{d^2y}{dx^2}\right)_1 < 0$ (1, 5) is a maximum | | | 1 |
| | $\left(\frac{d^2y}{dx^2}\right)_2 > 0$ (2, 4) is a minimum | | | 1 |
| | (Specific objectives: Sect 3: A4, A7-9, A11-17) | **2** | **6** | **6** |

**ADDITIONAL MATHEMATICS**
**PAPER 02**
**KEY AND MARK SCHEME**

**Question 6**

| Working | CK | AK | R |
|---|---|---|---|
| (a) $\int_1^2 (3x-1)^2\,dx = \int_1^2 \left(9x^2\ 6x + 1\right) dx$ | | | 1 |
| $= \left[3x^3 - 3x^2 + x\right]_1^2 = (24 - 12 + 2) - (3 - 3 + 1)$ | 1 | 1 | |
| $= 13$ | | 1 | |
| (b) $\int_0^{\frac{\pi}{2}} (5\sin x - 3\cos x)\,dx = \left[-5\cos x - 3\sin x\right]_0^{\frac{\pi}{2}}$ | 1 | 1 | |
| $= [-5(0) - 3(1)] - [-5(1) - 3(0)]$ | | | 1 |
| $= 2$ | | 1 | |
| (c) (i) $y = \int (2 - x)\,dx = 2x - \frac{x^2}{2} + C$ | 1 | | |
| At $\left(0, \frac{7}{2}\right)$ $C = \frac{7}{2}$ | | 1 | 1 |
| $y = 2x - \frac{x^2}{2} + \frac{7}{2}$ | | | |
| (ii) Area $= \int_0^5 \left(2x - \frac{x^2}{2} + \frac{7}{2}\right) dx = \left[x^2 - \frac{x^3}{6} + \frac{7x}{2}\right]_0^5$ | 1 | | 1 |
| Area $= \left(25 - \frac{125}{6} + \frac{35}{2}\right) = \frac{65}{3}$ units$^2$ | | 1 | |
| (Specific Objectives- Sect 3: B4-5, B7--10) | **4** | **6** | **4** |

**ADDITIONAL MATHEMATICS**
**PAPER 02**
**KEY AND MARK SCHEME**

## Question 7

(a)

(i) P (Statistics) = $\dfrac{28}{43}$  P (Physics) = $\dfrac{19}{43}$

P (Statistics and Physics) = $\dfrac{28}{43} + \dfrac{19}{43} - \dfrac{43}{43} = \dfrac{4}{43}$

(ii) P (Physics only) = $\dfrac{19}{43} - \dfrac{4}{43} = \dfrac{15}{43}$

(b)

(i) P (Red = 3 and Blue = 0) = $\dfrac{1}{16}$

(ii)

| | | | | |
|---|---|---|---|---|
| **3** | 0 | 3 | 6 | 9 |
| **2** | 0 | 2 | 4 | 6 |
| **1** | 0 | 1 | 2 | 3 |
| **0** | 0 | 0 | 0 | 0 |
| B \ R | **0** | **1** | **2** | **3** |

(iii)

| $t$ | 0 | 1 | 2 | 3 | 4 | 6 | 9 |
|---|---|---|---|---|---|---|---|
| P ($t$) | $\dfrac{7}{16}$ | $\dfrac{1}{16}$ | $\dfrac{1}{8}$ | $\dfrac{1}{8}$ | $\dfrac{1}{16}$ | $\dfrac{1}{8}$ | $\dfrac{1}{16}$ |

$a = \dfrac{7}{16}$,  $b = \dfrac{1}{8}$ and $c = \dfrac{1}{16}$

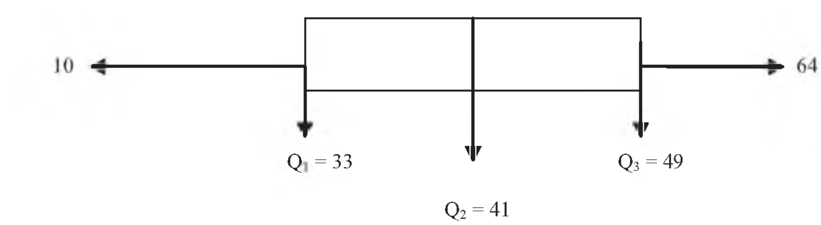
| CK | AK | R |
|---|---|---|
| 1 | 1 | 1 |
| | 1 | 1 |
| 1 | 1 | |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

**ADDITIONAL MATHEMATICS**
**PAPER 02**
**KEY AND MARK SCHEME**

**Question 7 cont'd**

(c) (i) $Q_1 = 33 \quad Q_2 = 41 \quad Q_3 = 49$

(ii)




The shape is symmetrical about the median

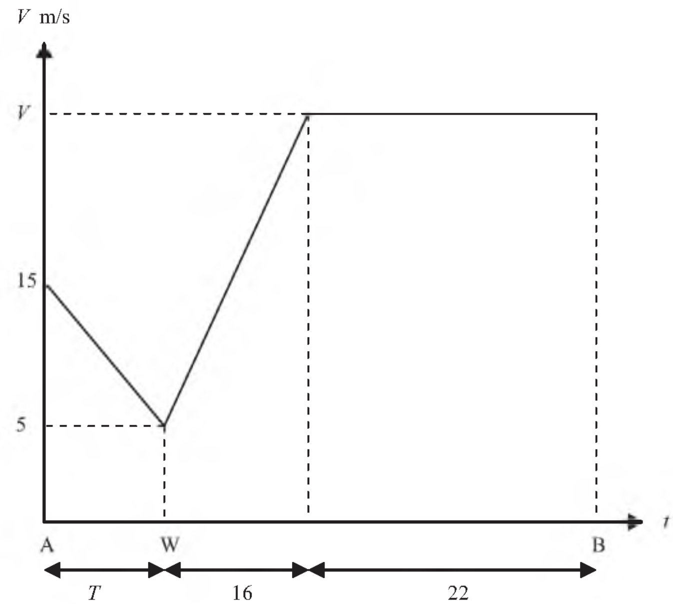
(Specific Objectives- Sect 4: A 4-6, B2-4, B9).

| CK | AK | R |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| | 1 | |
| **6** | **8** | **6** |

**ADDITIONAL MATHEMATICS**
**PAPER 02**
**KEY AND MARK SCHEME**

## Question 8

(a) (i)




| Working | CK | AK | R |
|---|---|---|---|
| (a) (i) Graph | 1 | 1 | 1 |
| (ii) Using area under the curve $\frac{1}{2}(15+5)(T)=120$ | 1 | | |
| $T=12$ | | 1 | |
| Total time = 12 + 16 + 22 = 50 secs | | | 1 |
| (iii) Using area under the curve | | | 1 |
| $120+\frac{1}{2}(V+5)\times 16+22V=1000$ | 1 | 1 | 1 |
| $V=28$ | | 1 | |

**ADDITIONAL MATHEMATICS**
**PAPER 02**
**KEY AND MARK SCHEME**

**Question 8**
**(cont'd)**

| | CK | AK | R |
|---|---|---|---|
| (b) (i) $v = 0 \Rightarrow 3t^2 - 30t + 72 = 0$ | | | 1 |
| $(t-6)(t-4) = 0$ | | 1 | |
| $t = 4$ secs, 6 secs | 1 | | |
| (ii) Distance $= \left\| \int_4^6 \left(3t^2 - 30t + 72\right) dt \right\|$ (below the $x$-axis) | | | 1 |
| $= \left\| \left[t^3 - 15t^2 + 72t\right]_4^6 \right\|$ | | 1 | |
| $= \left\|(216 - 540 + 432) - (64 - 240 + 288)\right\|$ | | | |
| = 4 metres | 1 | | |
| (iii) Distance in first 4 secs $= \int_0^4 \left(3t^2 - 30t + 72\right) dt$ | | | |
| = 112 metres | 1 | | |
| Distance between 6 secs and 7 secs | | | |
| $= \int_6^7 \left(3t^2 - 30t + 72\right) dt$ | | | |
| = 4 metres | | 1 | |
| Total distance = 112 + 4 + 4 = 120 metres | | 1 | |
| (Specific Objectives- Sect 4: C2 – 4) | **6** | **8** | **6** |

# CARIBBEAN EXAMINATIONS COUNCIL
HEADQUARTERS

## SECONDARY EDUCATION CERTIFICATE EXAMINATION

## ADDITIONAL MATHEMATICS

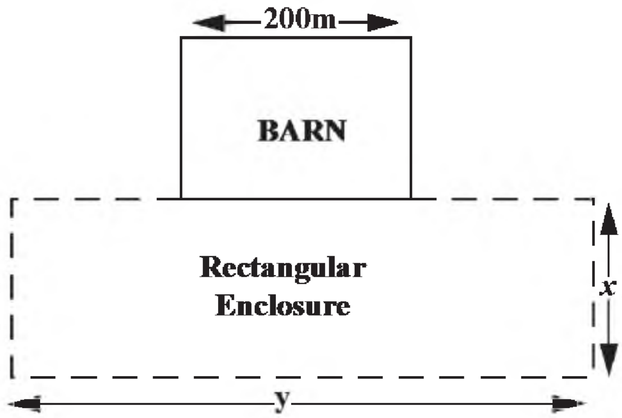
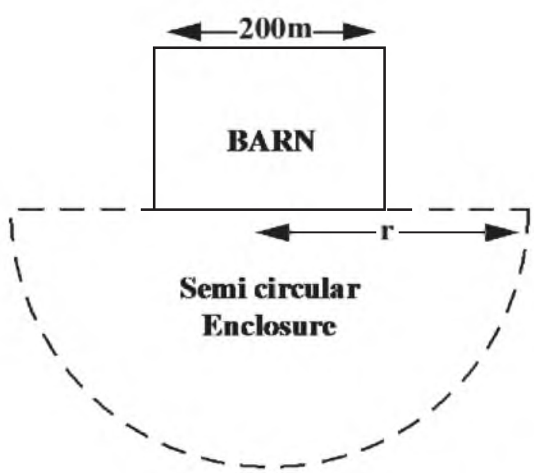
## PAPER 03/2

## SPECIMEN PAPER

## KEY AND MARK SCHEME

**MATHEMATICS**
**PAPER 03**
**KEY AND MARK SCHEME**

Question 1

| | | CK | AK | R |
|---|---|---|---|---|
| **(i)** | **Mathematical Formulation of the problem** | | | |
| | **Option I** | | | |
| |  | | | |
| | Let the rectangular enclosure have length *x m* and width *y* m. Since one side of | | | |
| | the barn of length 200 m is used in making the enclosure and an additional | | | 1 |
| | 800m of fencing is available then the perimeter of the enclosure is 200 + 800 = 1000 m | | | 1 |
| | The perimeter of the rectangle is $(2x + 2y)$ m and | 1 | | |
| | Area of the rectangle enclosure is $xy$ $m^2$ | | | |
| | Thus the problem can be formulated mathematically as | | | |
| | Maximise $A_1 = xy$ | | | |
| | Subject to $2x + 2y = 1000$ | 1 | | |
| | **Option II** | | | |
| |  | | | |

# MATHEMATICS
## PAPER 03
## KEY AND MARK SCHEME

| | CK | AK | R |
|---|---|---|---|
| Let the semicircular enclosure have radius r | | | 1 |
| Since half the circumference of a circle is $\pi$r and diameter is 2r, the problem is to find the Area $A_2$ of the semicircle when $\pi$r + 2r = 1000m | 1 | | |
| The solution lies in comparing the two areas Maximum $A_1$ and $A_2$ to see which is larger and recommend the larger option to the farmer. | | | 1 |
| (ii) **To show that for Option I a square maximizes the area:** | | | |
| Maximise $A_1 = xy$ (area)<br>Subject to $2x + 2y = 1000$ (perimeter) | | | |
| <u>Solution</u> | | | |
| $2x+2y = 1000 \longrightarrow y = \frac{1000-2x}{2} = 500 - x$ | | 1 | |
| Substituting in $A_1$ we obtain | | | |
| $A_1 = xy = x(500-x) = 500x - x^2$ | | | |
| To maximize $A_1$ with respect to $x$ we first find the stationary points: | | | 1 |
| $\frac{dA}{dx} = 500 - 2x$ | | | |
| $500 - 2x = 0 \longrightarrow 2x = 500$ | | 1 | |
| $\therefore$ x = 250 at maximum | | 1 | |
| (we need not find the 2nd derivative as there is only one solution but | | | |
| $d^2A/dx^2 = -2 \longrightarrow$ stationary point is a maximum) | | | |
| $\therefore y = 500 - x = 250$ | | 1 | |
| Since x = y the rectangle is a square | | | 1 |
| Maximum Area is $A_1 = 250^2 = 62\,500$ m$^2$ | | 1 | |
| (iii) **Option II Area** | | | |
| Perimeter of enclosure = $\pi r + 2r = 1000\ m$ | | | |
| $\therefore$ r = $\frac{1000}{\pi+2}$ = 194.49 m (radius of enclosure) | | | 1 |
| Area of semicircle is $A_2 = \frac{\pi}{2}\left(\frac{1000}{\pi+2}\right)^2$ | 1 | | 1 |
| This is the maximum area of the semicircle since r is fixed | | | |
| Area of semicircle is $A_2 = 59\,419$ m$^2$ | | 1 | |

**MATHEMATICS**
**PAPER 03**
**KEY AND MARK SCHEME**

| | | CK | AK | R |
|---|---|---|---|---|
| | **Recommendation** | | | |
| (iv) | Since 62500 > 59419, the square enclosure has greater area than the semicircular. I would therefore recommend Option I to the farmer, | | | **1** |
| | i.e. build a square enclosure of side 250 m | | | **1** |
| | | **4** | **6** | **10** |

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

# REPORT ON CANDIDATES' WORK IN THE CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION

**MAY/JUNE 2012**

## ADDITIONAL MATHEMATICS GENERAL PROFICIENCY EXAMINATION

## GENERAL COMMENTS

Additional Mathematics, the newest CXC subject offering at the CSEC level, was tested for the first time in the May/June 2012 examinations. The subject is intended to bridge a perceived gap between the CSEC General Mathematics and the CAPE Unit1 Pure Mathematics. The intent is to offer CSEC Mathematics students a more seamless transition to the advanced thinking and skills needed for CAPE Unit 1 Mathematics courses, although a good CSEC Mathematics student should still be able to meet the skills and thinking demands of the CAPE Unit 1 Mathematics course.

The examinations consist of three papers:

- Paper 01 — a 45 – item multiple choice paper;
- Paper 02 — a structured, 'essay-type' paper consisting of 8 questions;
- Paper 031 or Paper 032 — Paper 03 represents an School-Based Assessment (SBA) project component for candidates in schools, and Paper 032 an alternative to the SBA for out-of-school candidates.

The Additional Mathematics syllabus (CXC 37/G/SYLL 10) tests content in four main topic areas divided as follows: Section 1 — Algebra and Functions, Section 2 — Coordinate Geometry, Vectors and Trigonometry, Section 3 — Introductory Calculus, and Section 4 — Basic Mathematical Applications.

Paper 01 tests content from Sections 1, 2 and 3 of the syllabus. This paper carries 45 items which are weighted up to 60 for the overall examination. Paper 02 tests content from all four sections of the syllabus. This year the paper consisted of four sections, divided as described previously for the outline of the syllabus. Each section contained two problem-solving type questions. The questions in Sections 1, 2 and 3 were all compulsory. The two questions in these sections were worth a total of 28, 24 and 28 marks respectively. Section 4 also contained two questions, one on Data Representation and Probability and the other on Kinematics worth 20 marks each; candidates were to choose one question from this section. Paper 03 represents the SBA component of the examination. Candidates can do a project chosen from two project types, a mathematical modelling project (Project A) and a data handling/statistical analysis project (Project B). The SBA component is worth 20 marks. Alternatively, candidates can sit an alternative to the SBA paper, Paper 032, which consists of a more in-depth extended question from Sections 1, 2 and/or 3 of the syllabus. This paper carries 20 marks.

## DETAILED COMMENTS

### Paper 01 – Structured Essay Questions

This was a 45 - item paper covering Sections 1, 2 and 3 of the syllabus. A total of 1720 candidates sat this paper of the examination. Candidates scored a mean of 23.22, with standard deviation 8.77.

### Paper 02 – Structured Essay Questions

No formula sheet was provided for this sitting of the examination. However, in cases where it was suspected that candidates might have been affected by the absence of a formula sheet the mark scheme was adjusted so that candidates were awarded follow through marks if they used an incorrect reasonable formula, and were not penalized for this wrong formula. For future examinations, a formula sheet will be provided for papers 01 and 02.

**Section 1: Algebra and Functions**

Question 1

This question tested candidates' ability to:

- determine a composite function, its range and inverse;
- make use of the Factor Theorem;
- use the laws of indices to solve exponential equations in one unknown; and
- apply logarithms to reduce a relationship to linear form.

There were 1746 responses to this question. The mean mark was 5.72 with standard deviation 3.26. Thirty-nine candidates obtained full marks.

Candidates performed best on Parts (a) (i), (iii) and (b) which required them to determine the composite function *g(f(x))* of given functions *f(x)* and *g(x),* and the inverse of the composite function. In Part (a) (iii) some candidates experienced difficulty at the interchange step, that is,

1) (i) $g(f(x)) = x^3 + 6$; Let $y = g(f(x)$ (ii) $[*y^3 = x + 6*]$ commonly seen error

$y = \sqrt[3]{x+6}$

$\therefore [gf(x)]^{-1} = \sqrt[3]{x+6}$

Part (b) was generally well done by candidates. However, some of them did have difficulty here. The following common errors were seen:

- Substituting $x = 2$ into *f(x)* rather than $x = -2$;
- Some candidates who did correctly substitute $x = -2$ then made errors in their manipulation of the directed numbers;
- A few candidates attempted long division approach to solving this problem; however they very often encountered some difficulty with this.

Parts (a) (ii), (c) and (d) presented the most difficulty to candidates. In Part (a) (ii), which required candidates to state the range of the composite function, generally candidates were unable to do this, or stated discrete values for the range, that is, {6, 7, 14, 33}, not recognizing it as the infinite set of Real Numbers, $6 \leq g(f(x)) \leq 33$.

In Part (c), many candidates attempted to solve the given equation $3^{2x} - 9(3^{-2x}) = 8$ without (appropriate) substitution. Having not recognized a suitable substitution to use in order to transform the equation into polynomial form, candidates incorrectly took logarithms of each term in the equation. Some candidates were able to recognize that $y = 3^{2x}$ or $y = 3^x$ were suitable substitutions and quite often were able to derive an appropriate polynomial equation. Those who derived the quadratic equation were able to easily solve the indices equation. However, those who derived the quartic equation at times found it challenging to solve. A number of candidates who got to the solution stage of the problem did not recognize (or know) that the logarithm of a negative number did not exist.

In Part (d), although many candidates knew how to transform $x^3 = 10^{x-3}$ to logarithmic form, they did not use brackets and so wrote $3 \log_{10} x = x - 3 \log_{10} 10$, instead of the expected $3 \log_{10} x = (x - 3) \log_{10} 10$. Whilst candidates did not lose marks this time given the nature of the specific case here (base is the same as the number so that $\log_{10} 10$ is 1), teachers are asked to encourage their students to use brackets to avoid possible errors in the future. Another common error made was having reached to the stage of 3 $\log_{10} x = x - 3$, a number of candidates then went on to give the gradient as 1, seemingly not recognizing that they had to write the equation in terms of log $x$, that is. to divide through by 3 so that the coefficient of log $x$ was 1, i.e. $\log_{10} x = \frac{1}{3}x - 1$, so that the gradient of the linear form was $\frac{1}{3}$. Other common errors observed were:

- $\text{Log}_{10} x^3 = \log_{10} 10^{x-3}$
  $\text{Log}_{10} 3x = \log_{10} x - \log_{10} 3$

- $\text{Log}_{10} x^3 = \log_{10} 10x - \log_{10} 30$
  $3\log_{10} x = 10x - 30$

**Solutions**

(a) (i) $g(f(x)) = x^3 + 6$ (ii) $6 \le g(f(x)) \le 33$ (iii) $[gf(x)]^{-1} = \sqrt[3]{x-6}$

(b) $a = 20$

(c) $x = 1$

(d) (i) $\log_{10} x = \frac{1}{3}x - 1$ (ii) gradient $= \frac{1}{3}$

<u>Question 2</u>

This question tested candidates' ability to:

- make use of the relationship between the sums and products of the roots of quadratic equations;
- solve an inequality of the form $\frac{ax+b}{cx+d} \ge 0$;
- identify an AP, obtain an expression for its general term, find its sum to a finite term.

There were 1733 responses to this question. The mean mark was 5.64 with standard deviation 4.27. One hundred and four candidates obtained full marks

Candidates performed best on Part (c) where they had to recognize the problem as an Arithmetic Progression (AP) and find its first term and common difference.

Parts (a) and (b) presented the most difficulty to candidates. In Part (a), many candidates attempted to solve the quadratic equation whether by factorizing or using the formula, showing a seeming lack of knowledge about the sum and product of roots of quadratic equations. In Part (b), although some candidates were able to determine the boundary values for the inequality, they had difficulty with the direction of the inequality for these boundary values which would make the overall inequality 'true'. Although more rarely seen, there was generally more success among candidates who attempted a graphical solution to this question.

In Part (c), a common error observed related to candidates finding the 24th term of the AP ($T_{24}$) rather than the sum of the first 24 terms ($S_{24}$). Another common error observed was that some candidates could not correctly write the $S_n$ formula, commonly writing it as $S_n = \frac{1}{2}[a + (n - 1)d]$, instead of the expected $S_n = \frac{n}{2}[2a + (n - 1)d]$.

**Solutions**

(a) $\alpha^2 + \beta^2 = 4$

(b) $x > \frac{5}{2}$ and $x < -\frac{1}{3}$

(c) Total paid = $2100

**Section 2: Coordinate Geometry, Vectors and Trigonometry**

Question 3

This question tested candidates' ability to:

- find the centre of a given circle and showing that a given line passes through this centre;
- find the equation of a tangent to a circle at a given point;
- add and subtract vectors, find displacement vectors and find the magnitude of a vector.

There were 1715 responses to this question. One hundred and sixteen candidates obtained full marks.

Candidates performed best on Part (a) (ii) which required them to calculate the gradient of the radius of the circle (from two points), and hence determine the gradient and equation of the tangent to the circle's radius.

Candidates had difficulty with Part (a) (i) which required them to show that the line $x + y + 1 = 0$ passed through a given circle's centre. Although many could find the coordinates of the circle's centre, some candidates had difficulty in showing (that is, proving) that the given line passed through this centre, indicating a seeming lack of knowledge of what constituted a simple proof in this case. Candidates also had difficulty with Part (b), writing $\overrightarrow{BP}$ in terms of its position vectors, or correctly adding vectors to get $\overrightarrow{BP}$. A common error seen here was that $\overrightarrow{OP} = \overrightarrow{BP}$.

**Solutions**

(a) (ii) $y = \frac{4}{3}x + 11$

(b) (i) $\vec{BP} = \frac{3}{2}\mathbf{a} - \mathbf{b}$ (ii) $|\vec{BP}| = \frac{1}{2}$

Question 4

This question tested candidates' ability to:

- use the formulae for arc length and sector area of a circle;
- use the compound angle formula to evaluate a value for sin θ in surd form.

There were 1705 responses to this question. The mean mark was 4.20 with standard deviation 2.24. Two hundred and fifteen candidates obtained full marks.

Candidates performed best on Part (a), that is, finding the area and perimeter of a given shape, although some candidates did not seem to understand the instructions to give their answer in terms of π. For example, some candidates having arrived at the area of the figure as $16 + \frac{8\pi}{3}$ then went on to give a decimal answer. Other candidates mistakenly simplified this to $\frac{24\pi}{3}$. In Part (a) (ii), some candidates failed to recognize that one side of the square (or one radius of the sector) should not be included in the calculation for perimeter, and so added the whole perimeter of the square to the whole perimeter of the sector.

Part (b) presented difficulties for many candidates, in particular coming up with an appropriate split for $\frac{7\pi}{12}$; hence many candidates did not recognize the need for use of the compound angle formula. Some candidates who recognized the need to split then wrote it as $\cos\frac{\pi}{3} + \cos\frac{\pi}{4}$ for which no marks were awarded. Of note, a few candidates wrote that $\cos\frac{7\pi}{12} \equiv \cos\left(\frac{7\pi}{3} + \frac{7\pi}{4}\right)$ and used the fact that 2π is one revolution to come to the correct compound angle $\cos\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$. The concept of exactness also appeared to be lost on many of the candidates who proceeded to write the solution, $\frac{\sqrt{2}-\sqrt{6}}{4}$ as a decimal, or some other incorrect simplification, such as $-\frac{\sqrt{4}}{4}$.

Part (c) was not marked. However, approximately $\frac{1}{3}$ of the candidates who attempted this part of the question knew that $\sec x = \frac{1}{\cos x}$.

**Solutions**

(a) (i) $16 + \frac{8\pi}{3}\text{ m}^2$ (ii) $16 + \frac{4\pi}{3}\text{ m}$

(b) $\frac{\sqrt{2}-\sqrt{6}}{4}$

**Section 3: Introductory Calculus**

<u>Question 5</u>

This question tested candidates' ability to:

- differentiate a quotient made up of simple polynomials;
- determine the equations of tangents and normal to curves;
- make use of the concept of derivative as a rate of change.
-

There were 1712 responses to this question. The mean mark was 5.89 with standard deviation 4.47. One hundred and thirty-three candidates obtained full marks.

In this question, candidates performed best on the mechanics of differentiation. Many though could not state the correct quotient rule for differentiation, and so performed the mechanics of differentiation in Part (a) as if for a product rule. And, having performed the mechanics of differentiation correctly, a marked number of candidates then could not apply the distributive law correctly with a negative 1 multiplier to expand the brackets, i.e. $-1(3x + 4)$ was often expanded as $-3x + 4$.

Part (b) was generally reasonably done. Some candidates experienced difficulty with the meaning/significance of $\frac{dy}{dx}$, in that having found it for the given curve, they could not translate it into finding the value of the gradient at the given point (2, 10). That said, making use of a value for gradient to find the equation of the tangent and the equation of the normal was fairly widely known.

Approximately 50 per cent of candidates did not appear to know the topic 'Rates of Change' and so could not do Part (c). Others, having determined that the area, $A$, of the square ($s$) could be written as $A = s^2$ then went on to differentiate this as $\frac{dy}{dx} = 2s$ instead of $\frac{dA}{ds} = 2s$, showing a lack of understanding of the meaning of differentiation as rate of change. Candidates' difficulty with notation was especially evident in this part of the question; very few candidates wrote their differential equations in terms of $dt$.

**Solutions**

(a) $\frac{-10}{(x-2)^2}$ (b) (i) $y = 17x - 24$

(ii) $10y = x + 172$

(c) $\frac{dA}{dt} = 40\text{ cm}^2\text{s}^{-1}$

Question 6

This question tested candidates' ability to:

- evaluate the definite integral of a function [of the form $(ax \pm b)^n$, $n \neq -1$];
- determine the equation of a curve given its gradient function and a point on the curve;
- find the area of a region bounded by a trigonometric curve and the $x$-axis;
- find the volume of revolution of the solid formed when a curve of degree 2 is rotated about the $x$-axis.

There were 1655 responses to this question. The mean mark was 6.23 with standard deviation 4.61. One hundred and two candidates obtained full marks.

Candidates performed best on Part (b), although some lost marks as they did not include or failed to find a value for the constant of integration. Part (c) was also relatively reasonably done, although some candidates experienced difficulties in correctly evaluating cos $\theta$. In Part (a), in their integration of $(16 - 7x)^3$, although many candidates could perform the mechanics of integration to obtain 4 in the denominator, many candidates failed to multiply the coefficient of $x$ in the polynomial by the 4 to obtain −28 for the denominator. A few candidates expanded the polynomial in order to integrate it.

Part (d) presented the most difficulty for candidates. Many candidates could set up the problem as $\pi\int_0^3 (x^2+2)^2\,dx$, although some did forget the '$\pi$' in their formula, but a number of candidates were unable to expand $(x^2 + 2)^2$ correctly, very often obtaining $x^4 + 4$.

**Solutions**

(a) $\dfrac{935}{4}$ (b) $y = \dfrac{3x^2}{2} - 5x + 4$ (c) $\dfrac{2\sqrt{3}+3}{2}$ units$^2$

(d) $96.6\,\pi$ units$^3$

**Section 4: Basic Mathematical Applications**

Question 7

This question tested candidates' ability to:

- make use of the laws of probability, specifically the sum of all probabilities in a sample and the addition rule;
- calculate the probability of an event occurring;
- construct and use a tree diagram and calculate conditional probability.

Candidates performed best on Part (a). Many used their knowledge of Sets and Venn Diagrams from CSEC Mathematics to represent the given problem and successfully solve it. However, for candidates who used a formula method to solve, a common error observed was forgetting to subtract the intersection $P(L \cap D)$ (that is, owned both a laptop and a desktop computer) from the $P(L) + P(D)$.

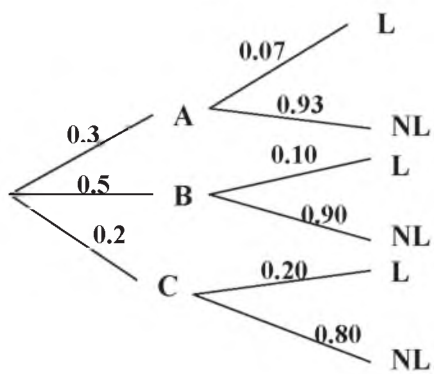
Parts (b) and (c) presented the most difficulty for candidates. In Part (b), a common error observed related to candidates presenting a solution representing sampling with replacement, although the question stated that the items were withdrawn without replacement.

In Part (c) (i), many candidates had difficulty drawing the tree diagram and it was clear that some did not know what this was as they drew diagrams of actual trees, or drew stem and leaf diagrams. Of those who did know what to do, many could only correctly draw the first branch, and had problems completing the second branches for 'late/not late'. Two common errors observed were candidates who correctly wrote the probability of a taxi being late from garage A as $P(L) = 0.07$ but then calculated $P(\bar{L}) = 0.23$, and candidates who incorrectly wrote the probability of a taxi being late from garage A as $P(L) = 0.7$ instead of $P(L) = 0.07$ as given. In Part (c) (ii) b, candidates recognized that conditional probability was involved but did not know the formula for conditional probability; they knew that some sort of division was necessary but did not know what to divide by, with many not using the probability they had computed in Part (c) (ii) a) as the denominator here.

**Solutions**

(a) 17% (b) (i) $\frac{1}{20}$ (ii) $\frac{1}{2}$

(c) (i)




(c) (ii) (a) 0.111 (b) $\frac{40}{111}$

Question 8

This question tested candidates' ability to:

- draw and interpret and make use of velocity-time graphs;
- apply rates of change to the motion of a particle.

There were 879 responses to this question. The mean mark was 6.96 with standard deviation 5.02. Seven candidates obtained full marks.

Candidates performed best on Parts (a) (i) and (ii) which required them to draw a velocity-time graph from given information, and also to determine the distance.

Part (a) (iii) however presented the most difficulty for candidates. This part dealt with a second car starting 3 seconds after the first car from the same starting point, moving at a constant (faster) velocity and meeting the first car sometime later. Most candidates did not know that the distances covered would have been the same at the meeting point, or that the second car would have taken 3 seconds less than the first car to reach that point. Candidates' experience of difficulty with Part (b) related mainly to notations and their failure to incorporate the constant of integration into their solutions.

**Solutions**

(a) (ii) 264 m (iii) 6 seconds

(b) (i) velocity = 32 $ms^{-1}$ (ii) displacement = $\frac{52}{3}$ m

**Paper 031 - School -Based Assessment (SBA)**

Generally, many of the projects submitted were of a high quality and related to real-world situations. Many showed a high level of ingenuity, and were obviously scenarios in which students were interested (for example, modelling the trajectory of a basketball to maximize the number of points that could be gained from free throws). A number of the submissions showed evidence that students had applied a high degree of effort to the task, understood the focus of their topic selection and had conducted the requisite research thoroughly. Projects were generally well presented. Marks were generally in the range of 14 to 20, with many of the sample submissions receiving the full score of 20 marks.

There were nonetheless a number of observed weak areas related to sample submissions. These included:

- Some SBA project titles and aims/purpose were not clear, not concise and not well defined. Some titles were too vague, others too wordy. In some cases, students simply restated the project's title as its aim/purpose;
- In a number of cases, the project's aim did not relate to the given title and sometimes not linked to any syllabus objective;
- In some cases for Project B, students did not identify their variables which would be a key aspect of this project type related to the analysis and discussion of the results;
- In some cases for both Projects A and B assumptions were not always stated;
- Some students' data collection method(s) were flawed, with the resulting negative impact on analysis and findings;
- It appeared that all (or most) of the students at some centres conducted their research using the same topic or a limited number of topics. Although there is nothing untoward with this approach, teachers must be aware that the probability of students presenting identical results, analyses and conclusions is very remote. In essence, students must demonstrate individuality in their work;
- In presenting their data or mathematical formulations, many students were short on explanations of what was happening so it was sometimes difficult to follow their presentations;
- Many students had difficulty presenting an appropriate, coherent and full analysis of their data, hence finding(s) and conclusion(s) were deficient;
- The majority of students did not incorporate or suggest any future analysis in their submissions;
- Some students failed to adequately connect their finding(s), discussion and/or conclusion(s) to their original project aim(s)/purpose, their data collected and their analysis of the data. In some cases, the conclusions stated did not arrive out of the presented findings;
- A few submissions appeared to be downloaded from the internet, and there were cases where submitted samples were blatantly plagiarized from CXC/CSEC Additional Mathematics material. Teachers must be alert to any suspected plagiarism. Plagiarism must not be condoned and marked accordingly;

- Some students presented projects that were more applicable to the sciences with an inadequate application of Additional Mathematics concepts;
- Whilst projects were generally well presented, a marked number of them showed deficiencies in grammar, spelling and punctuation.

Other general issues encountered in moderating the SBA samples included:

- There was no CSEC Additional Mathematics rubric/mark scheme accompanying the samples submitted by some centres;
- Some teachers created their own rubric or made adjustments to the CSEC Additional Mathematics rubric;
- Some projects were incorrectly categorized and assessed, that is, a Project A being labelled and assessed as a Project B, and vice versa. This did create some problems as for example, Project B requires the collection and analysis of data from an experimental-type activity.

The following recommendations hold for continued improvement in this aspect of the Additional Mathematics examinations:

- All projects should have a clear and concise title, and well defined aim(s) or purpose;
- Where possible the SBA should identify with authentic situations;
- The variables that are being used or measured (Project B) must be clearly stated and described. Variables can be controlled, manipulated and responding;
- The type of sample and sample size if relevant must be clearly stated;
- Teachers must ensure that projects which integrate other subject areas utilize concepts as contained in the CSEC Additional Mathematics syllabus;
- If students collect their data in a group setting, they must demonstrate their individual effort in relation to analysis (interpretation) and finding(s)/conclusion(s);
- Projects involving dice or playing cards must be more expansive so that students can present a more in-depth analysis of the topic under consideration;
- As good practice, students should be encouraged to cite all sources and insert a reference/bibliography page;
- Teachers should guide students using the assessment criteria found in forms 'Add Math 1 – 5A' and 'Add Math 1 – 5B' which are both available on the CXC website. Teachers can give their students the rubric as a means of guidance in developing their projects.

**Paper 032 - Alternative to School-Based Assessment (SBA)**

This paper tested candidates' ability to:

- formulate a mathematical problem and derive values to maximize its solution;
- make use of the laws of indices and logarithms to evaluate a specific $n^{th}$ value of an AP.

There were 54 responses to this paper. The mean mark was 5.24 with standard deviation 4.22. The highest mark awarded was 18/20.

In Part (a), candidates were able to formulate the mathematical problem as directed. However, a number of candidates lost marks for not finding the second derivative $\frac{d^2A}{dx^2}$ (for both sports clubs) and confirming that it was a maximum. Candidates also made various arithmetic errors in their solutions so few were able to come up with the expected dimensions for both sports clubs.

All candidates who attempted Part (b) approached a solution using the given information that the series was an AP. A number of candidates were able to get one expression for the common difference $d$ by subtracting two consecutive terms given. However, many failed to obtain a second expression for $d$ and so could not equate these two to get an expression for $a$ in terms of $b$. Many did not realize that they needed to substitute for $a$ and so ended up with a final expression in $ab$, which, in some cases even though correct, would not have given them the requested value for $n$.

**Solutions**

(a) (i) Maximize $A = 6xy$ Subject to $9x + 8y = 600$

(ii) Maximum area Q = 7500 $m^2$ (iii) Maximum area P = 22 500 $m^2$

(b) $n = 112$

***Appendix: Suggested Solutions to Selected Question Parts in Paper 02***

1(c) Solve $3^{2x} - 9(3^{-2x}) = 8$ [1]

Let $3^{2x} = m$; Substitute into [1] gives

$m - \dfrac{9}{m} = 8$

$m^2 - 9 - 8m = 0$

$m^2 - 8m - 9 = 0$

$(m - 9)(m + 1) = 0$

$m = 9$ OR $m = -1$

that is, $3^{2x} = 9$ OR $3^{2x} = -1$ (INVALID)

From $3^{2x} = 9$

$2x \log 3 = \log 9$

$2x = \dfrac{\log 9}{\log 3} = 2$

$x = 1$

2(b) $\dfrac{2x-5}{3x+1} > 0$

For LHS > 0, numerator AND denominator must BOTH have the same sign

For $2x - 5 > 0$ AND $3x + 1 > 0$

$x > \dfrac{5}{2}$ AND $x > \dfrac{-1}{2}$

Both true when $x > \dfrac{5}{2}$ [1]

AND for $2x - 5 < 0$ AND $3x + 1 < 0$

$x < \dfrac{5}{2}$ AND $x < \dfrac{-1}{2}$

Both true when $x < \dfrac{-1}{2}$ [2]

From [1] AND [2], range of values when $x > \dfrac{5}{2}$ AND $x < \dfrac{-1}{2}$

5(c) $\dfrac{ds}{dt} = 4 \text{ cm s}^{-1}$

$A = s^2$; $\dfrac{dA}{ds} = 2s$

$\dfrac{dA}{dt} = \dfrac{dA}{ds} \times \dfrac{ds}{dt}$

$= 2s \times 4 = 8s$

When $s = 5$, $\dfrac{dA}{ds} = 8 \times 5 = 40 \text{ cm}^2 \text{ s}^{-1}$

7(b) Let R = red marble, B = blue marble, K = black marble

(i) P(all of same colour) = P($RRR$) + P($BBB$) + P($KKK$)

$$= \frac{4}{10} \times \frac{3}{9} \times \frac{2}{8} + \frac{3}{10} \times \frac{2}{9} \times \frac{1}{8} + \frac{3}{10} \times \frac{2}{9} \times \frac{1}{8}$$

$$= \frac{1}{20}$$

(ii) P(exactly one Red) = P($R\overline{R}\,\overline{R}$) × 3

$$= \frac{4}{10} \times \frac{6}{9} \times \frac{5}{8} \times 3$$

$$= \frac{1}{2}$$

7(c) (ii) (a) P($L$) = P($A \cap L$) + P($B \cap L$) + P($C \cap L$)

$= 0.3 \times 0.07 + 0.5 \times 0.1 + 0.2 \times 0.2$

$= 0.021 + 0.05 + 0.04$

$= 0.111$

(b) $$P(C/L) = \frac{P(C \cap L)}{P(L)}$$

$$= \frac{0.2 \times 0.2}{0.111}$$

$$= \frac{0.04}{0.111}$$

$$= \frac{40}{111}$$

C A R I B B E A N  E X A M I N A T I O N S  C O U N C I L

# REPORT ON CANDIDATES' WORK IN THE CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION

## MAY/JUNE 2013

## ADDITIONAL MATHEMATICS

## GENERAL PROFICIENCY EXAMINATION

## GENERAL COMMENTS

Additional Mathematics, the newest subject offering at the Caribbean Secondary Education Certificate (CSEC) level, was tested for the second time in the May/June 2013 examinations. The subject is intended to bridge a perceived gap between the CSEC General Mathematics and the CAPE Unit 1 Pure Mathematics. The intent is to offer CSEC Mathematics students a more seamless transition to the advanced thinking and skills needed for CAPE Unit 1 Mathematics courses, although a student who masters the content and skills set out in the CSEC Mathematics syllabus should be ready to meet the skills and thinking demands of the CAPE Unit 1 Mathematics course.

The examinations consist of four papers:

- Paper 01 – a 45-item multiple choice paper
- Paper 02 – a structured, 'essay-type' paper consisting of eight questions
- Paper 031– a School-Based Assessment (SBA) project component for candidates in schools
- Paper 032 – an alternative to the SBA for out-of-school candidates.

Questions in the examination targeted the content and specific objectives of the Additional Mathematics syllabus (CXC 37/G/SYLL 10) with the aim of ensuring appropriate breadth and depth of syllabus coverage.

Paper 01 tests content from Sections 1, 2 and 3 of the syllabus and consists of 45 items. Paper 02 tests content from all four sections of the syllabus. This year the paper consisted of four sections and each section contained two problem-solving type questions. The questions in Sections 1, 2 and 3 were all compulsory. The two questions in these sections were worth a total of 28, 24 and 28 marks respectively. Section 4 also contained two questions, one on Data Representation and Probability and the other on Kinematics, each worth 20 marks. Candidates were required to choose one question from this section. Paper 031 represents the SBA component of the examination. Candidates can do a project chosen from two project types, a mathematical modelling project (Project A) and a data handling/statistical analysis project (Project B). The SBA component is worth 20 marks. Alternatively, private candidates can sit an alternative paper to the SBA, Paper 032, which consists of an in-depth, extended question from Sections 1, 2 and/or 3 of the syllabus. This paper carries 20 marks.

This year saw a 69 per cent increase in candidates registered for the examination, up from 1720 candidates in 2012 to 3100 in 2013.

This year, a formula sheet was included as part of the question paper on Papers 01 and 02.

## DETAILED COMMENTS

### Paper 01 – Multiple Choice

This was a 45-item paper covering Sections 1, 2 and 3 of the syllabus. The mean score on this paper was 35.25, with standard deviation of 13.31, compared with 30.96 and 11.69 in 2012.

### Paper 02 – Structured Essay Question

This paper consisted of 8 questions, of which questions 1 – 6 were compulsory. Candidates had to choose either Question 7 or Question 8. The total possible score on this paper was 100 marks. The mean and standard deviation for this paper were 47.82 and 27.2 respectively, compared to 37.32 and 22.95 in 2012.

**Section 1: Algebra and Functions**

Question 1

This question tested candidates' ability to:

- make use of the Factor Theorem and factorize a polynomial of degree 3, obtaining all its linear factors;
- determine the inverse of a given function and the composite of two functions;
- use logarithms to solve equations of the form $a^x = b$; simplify expressions using Laws of Logarithms.

There were approximately 2900 responses to this question. The mean mark was 8.02 and the standard deviation was 3.99. Three hundred and forty-four candidates obtained full marks (14 marks).

Candidates performed best on Parts (a) and (b) (ii), with many candidates earning full marks. Part (a) required candidates to show that a given linear expression was a factor of a given polynomial and then go on to find the other linear factors of the polynomial. Most candidates were able to do so either by using the Factor Theorem or via the longer process of long division. More often than not, candidates were then able to successfully factorize the polynomial expression completely. However, the quadratic factor $x^2 - 5x + 6$ was commonly incorrectly factorized as $(x - 1)(x - 6)$. In Part (b) (ii) candidates were required to find an expression for the composite of two functions. Generally, candidates were able to make the correct substitution, although some then made errors in the simplification process. Others still, having correctly simplified and obtained the correct result of $\frac{2x+1}{x+3}$, then proceeded to erroneously cancel out the $x$ in both the numerator and the denominator.

Part (b) (i) and Part (c) presented the greatest difficulty to candidates. In Part (b) (i), which required candidates to obtain an expression for an inverse function, some candidates, having interchanged the variables $x$ and $y$, experienced difficulties in making $y$ the subject of the formula. Additionally, some candidates did not write their final obtained expression in terms of $f^{-1}(x)$, and simply left it in terms of $y$. In Part (c), although most candidates knew they needed to take logarithms, they made many errors in applying the mechanics of this to the given problem. For example, in bringing down the indices they did not use brackets, and so made errors in the expansion of the expressions obtained. A common error seen was:

$\lg[5^{3x-2}] = \lg[7^{x+2}]$
$3x - 2\lg 5 = x + 2\lg 7$
$2x = 2\lg 7 + 2\lg 5.$

With respect to candidates' responses on this question, there is consolidation needed in the following areas:

- The correct transposition of equations containing more than one variable. For example, transpose for $y$, $xy + 2x = 2y - 1$.
- Students must be alert to the fact that when finding $f^{-1}(x)$, the final result must contain $f^{-1}(x)$ equal to some expression in $x$.
- Students **must not** consider as equivalent, the *factors of a polynomial expression* and the *roots of a polynomial equation*.
- The correct application of the laws of logarithms. Examples of common misconceptions presented were: $(\log A)(\log B) = \log A + \log B$ ; $\log\left(\frac{A}{B}\right) = \frac{\log A}{\log B}$

**Solutions**: (a) (ii) $(x-3), (x-2)$

(b) (i) $f^{-1}(x) = \dfrac{2x+1}{2-x}$ (ii) $fg(x) = \dfrac{2x+1}{x+3}$

Question 2

This question tested candidates' ability to:

- express a quadratic function in the form a(x + h) + k where a, h and k are constants; and determine the function's minimum value;
- find the solution set of a quadratic inequality;
- identify geometric series and calculate the sum to infinity of geometric series.

There were 2959 responses to this question. The mean mark was 6.93 and the standard deviation was 3.59. Ninety-one candidates obtained full marks (14 marks).

Candidates performed best on Part (b) of this question, which required them to find the solution set of a quadratic inequality. Most candidates correctly factorized the quadratic and were able to identify the critical values as $\dfrac{-5}{2}$ and 1. However, correctly *stating* the solution to satisfy the inequality proved to be a challenge for a number of candidates, even in cases where they were able to correctly identify the regions which satisfied the inequality in a sketch.

Parts (a) and (c) presented some difficulty to candidates. In Part (a), candidates were required to write a given quadratic expression in the form $a(x + h)^2 + k$. Approximately 50 per cent of the candidates attempted this question by the method of completing the square, with some candidates encountering difficulties. Some of the candidates used the strategy involving the identity to find the solution as follows:

$$3x^2 + 6x - 1 \equiv a(x + h)^2 + k \rightarrow 3(x + 1)^2 - 4 \quad \text{where h = 1 and k = −4}$$

Deductions: (i) Minimum value of function = −4 (ii) Value of $x$ at minimum = −1

A few candidates attempted to solve for roots of function and abandoned the solution without completing the square.

In Part (c) most candidates were unable to identify two series from the given series, and generally the strategy of separating a given series into two appeared to be unknown to most candidates. Most candidates who attempted the question ignored the note given at the end of the question, and proceeded to find the sum to infinity as if for one series. Very few candidates attempted to separate the series into two GPs. There were also a few candidates who did not seem to know which formula to use to find the sum to infinity, and rather found the sum of the first two terms.

**Solutions**: (a) (i) $3(x + 1)^2 - 4$

(ii) Minimum value $f(x) = -4$

(iii) Value of $x$ at minimum $= -1$

(b) Solution set: $\{x: x \leq \frac{-5}{2}\} \cup \{x: x \geq 1\}$

(c) $\frac{2}{5}$

**Section 2: Coordinate Geometry, Vectors and Trigonometry**

Question 3

This question tested candidates' ability to:

- write the equation of a circle, find the radius of a given circle; and find the equation of a tangent to a circle at a given point;
- apply properties of perpendicular vectors;
- derive a unit vector.

Candidates performed best on Part (a) (i), and Parts (b) and (c) of this question. In Part (a) (i) many candidates were able to write the equation of the circle (without consideration of the radius), and responses of $x^2 + y^2 - 4x - 2y + K = 0$ or $(x - 2)^2 + (y - 1)^2 = r^2$ were often seen. In Part (a) (ii) candidates also knew how to find the gradient of the radius and the fact that the tangent is perpendicular to the radius (normal).

In Part (b), a significant number of the candidates either knew that the dot product of perpendicular vectors is zero ($\vec{OP} \cdot \vec{OQ} = 0$) or used the formula to find the angles between two vectors and substituted 90° for this angle. In Part (c), the majority of the candidates knew how to find $\overrightarrow{AB}$, either from $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$ or $\overrightarrow{AB} = -\overrightarrow{OA} + \overrightarrow{OB}$. However, they made mistakes in writing down the vectors or in changing the signs, and responses of $-2i - 5j + 3i - 7j = i - 12j$ or $2i + 5j + 3i - 7j = 5i - 2j$ were often seen. Candidates also knew how to find the modulus of a vector.

Candidates did have difficulty with part of Part (a) (i), specifically to determine r or K in the formula. Many did not seem to know how to do this, or that they needed to do this, and $(x - 2)^2 + (y - 1)^2 = 0$ was often seen. In Part (b), correctly computing an expression for the dot product proved to be a problem and $10\lambda + 80$ was often seen. Additionally, some candidates confused the concepts of perpendicularity of lines and perpendicularity of vectors, thus equating the dot product to $-1$ was seen. In Part (c) finding the unit vector proved to be problematic even in cases where the vector $\overrightarrow{AB}$ and its modulus were correctly found.

With respect to candidates' responses in this question, the following areas of consolidation are needed:

- Candidates should be taught to derive expressions rather than memorize them. The formula $\frac{dy}{dx} = \frac{-(g + x)}{f + y}$ was used by some candidates to find the gradient of the tangent; however some of them did not know that $g$ in their expression was half of $h$ from the given formula for the equation of a circle and $f$ was half of $g$ from the given equation of the circle.

- Candidates should also be encouraged to use techniques appropriate to the topic being tested; for example, in the coordinate geometry question some candidates tried to use calculus to determine the gradient of the tangent. While this technique was a legitimate one, only a few of them were able to correctly do it. The concept of the product of gradients being equal to −1 should be used for perpendicularity of lines in coordinate geometry, while for vectors, equating the dot product to zero should be the preferred method of establishing perpendicularity.

- When the coordinates of the centre of a circle and a point on the circumference are given, candidates should be encouraged to use the formula $(x - a)^2 + (y - b)^2 = r^2$, where $(a, b)$ are the coordinates of the centre and $r$ is the radius.

- The definition of a unit vector and how to determine a unit in the direction of a given vector should be emphasized.

**Solutions**: (a) (i) $x^2 + y^2 - 4x - 2y - 95 = 0$

(ii) Equation of the line l: $y - 3 = \frac{-4}{3}(x - 10)$

(b) $\lambda = 8$

(c) $\overrightarrow{AB} = \frac{1}{13}(5i - 12j)$

Question 4

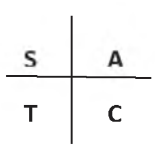
This question tested candidates' ability to:

- use the formulae for arc length and sector area of a circle;

- find solutions of simple trigonometric equations for a given range, which involve the use of $\cos^2\theta + \sin^2\theta \equiv 1$.

- use the compound angle formula for tan $\theta$.

Candidates performed best on Part (b), that is, solving a given trigonometric equation. Candidates were able to state the necessary trigonometric identity and perform the required substitution, factorize the quadratic equation obtained, identify the range of the trigonometric function and so correctly solve the trigonometric equation for the given range. Some candidates, though, having found the principal angle/value, did have difficulty calculating the other angles that would fit the solution in the given range. Candidates readily drew:




but seemed unsure as to how and why this aid was developed. Teachers are urged to explore why each of the trigonometric ratios is positive or negative.

Part (a) presented the most difficulties for candidates. A number of candidates incorrectly stated the formula for the perimeter of a sector, most often only writing this in terms of the arc length and not including $2r$. Many then attempted to equate coefficients in finding a value for $r$ instead of using an algebraic method, and so did not obtain a correct value for $r$, nor the sector's area, as asked for in the question. In Part (c), whilst most candidates were able to identify and use $\frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$ ≡ tan ($A$ ± $B$), some candidates had difficulty interpreting that the ± and the ∓ signs, and so often wrote: tan ($A$ – $B$) = $\frac{\tan A - \tan B}{1 - \tan A \tan B}$

Teachers are encouraged to have students learn the formulae and to not rely solely on the formula sheet.

**Solutions**: (a) Area = $\frac{25\pi}{12}$ m$^2$

(b) $\theta = 210^{\circ}$ and $330^{\circ}$

(c) $\alpha = 45^{\circ}$

**Section 3: Introductory Calculus**

Question 5

This question tested candidates' ability to:

- differentiate a simple polynomial made up of $x^n$, use the concept of stationary points; calculate the second derivative of polynomials; interpret the significance of the sign of the second derivative; and use the sign of the second derivative to determine the nature of stationary points;
- differentiate sin $x$, differentiate products of simple polynomials and trigonometric functions; apply the chain rule in the differentiation of composite functions.

In this question candidates performed best on Part (a) (i) and well on Part (a) (ii). In Part (a) (i) the concept of stationary points appeared to be familiar to most candidates, and approximately 70 per cent were able to successfully identify the coordinates of the stationary points. Incorrect solutions were obtained by candidates who equated the function rather than the first derivative to zero to obtain the coordinates of the stationary points. Common errors in algebraic manipulations involved solving $3x(x - 2) = 0$, and writing $3x = 0$, $x = -3$. In Part (a) (ii) most candidates were able to correctly find the second derivative and use the x-values from Part (a) (i) to deduce the nature of the stationary points. However, a common error seen was equating the second derivative to 0 and solving for $x$ in order to determine the nature of the stationary points.

Part (b) presented the most difficulty to candidates. It required them to simultaneously use the product and the function of a function rule in differentiating a combination trigonometric and polynomial expression. A number of candidates stopped at one term of the product rule and differentiation of the term in $x$ in the brackets was not done. Additionally, many candidates incorrectly stated the derivative of sin $x$ as –cos $x$. Finally, some candidates had problems simplifying the expression obtained, with a number of them attempting to expand the algebraic term as simplification of the result.

**Solutions**: (a) (i) Coordinates stationary points = (0, 2) and (2, –2)

(ii) $\frac{d^2y}{dx^2} = 6x - 6$; (0, 2) is a maximum point, (2, –2) is a minimum point

(b) $(5x + 3)^2\,[(5x + 3)\cos x + 15 \sin x]$

Question 6

This question tested candidates' ability to:

- use the rules of simple integration;
- integrate simple trigonometric functions and compute definite integrals;
- formulate the equation of a curve, given its gradient function and points on the curve; find the area of a finite region in the first quadrant bounded by a curve and the $x$- and $y$-axes;

There were 2800 responses to this question. The mean mark was 6.16 and the standard deviation was 4.67. Three hundred and nineteen candidates obtained full marks (14 marks).

Part (a) presented the most difficulty to candidates. In Part (a) a number of candidates did not remember to include the arbitrary constant in integrating the given function. In Part (c) some candidates could not obtain the correct equation of the curve. Even among those who knew to integrate the gradient function to obtain this equation, some did not include a constant of integration and hence obtained an incorrect equation. Additionally, a minority had difficulty obtaining the limits between which they should integrate.

**Solutions:** (a) $\frac{5}{3}x^3 + 4x + C$

(b) $-2$

(c) Area = $26\frac{2}{3}$ units$^2$

**Section 4: Basic Mathematical Applications**

Question 7

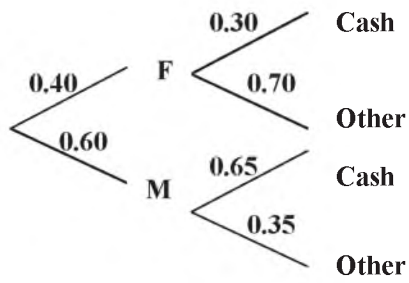
This question tested candidates' ability to:

- construct and use a tree diagram to solve problems involving probability; calculate conditional probability; calculate classical probability;
- state the relative advantage of a stem and leaf diagram versus a box and whisker plot to display data; construct a stem and leaf diagram; determine quartiles and measures of central tendency from stem and leaf diagram.

Candidates performed best on Part (a) (i) of this question, which required them to complete the construction of a tree diagram. Candidates knew that the sum of each branch equates to 1. However, having drawn the tree diagram, many candidates did not use it to answer Parts (a) (ii) and (iii). Part (a) (ii), on conditional probability, was badly done, and it appeared that most candidates did not even realise that the question required the use of conditional probability. In Part (a) (iii), most candidates were able to get the probability for Event V, and also to say which was the more likely event.

Parts (b) (i), (ii), (iv) and (v) presented the most difficulty to candidates. In Part (b) (i), which asked candidates to state an advantage of a stem and leaf diagram versus a box and whiskers plot to display the given data, a number of candidates stated their personal preference rather than an advantage, and wrote, for example, 'the stem and leaf is easier to draw and understand'. In Part (b) (ii), which required candidates to construct the stem and leaf diagram, a number of candidates drew tree diagrams. A number of candidates who did draw a stem and leaf diagram lost a mark for not including a key to the diagram. In Part (b) (iv), candidates did not know the formula for semi-interquartile range, or did not know how to work this out. In many cases they could not correctly find the lower and upper quartiles, and many did not know to divide this difference by two. In Part (b) (v), some candidates mistook the sample space for the marks. Many other candidates did not recognize the sample space being reduced, and so could not correctly give the probability of the second student scoring less than 50 marks on the exam.

**Solutions**: (a) (i)




(ii) P(Cash) = 0.51

(iii) Event T more likely

(b) (i) All data values are retained; shape of distribution easily seen

(ii)

| **Stem** | **Leaf** |
|---|---|
| 4 | 0 1 5 |
| 5 | 0 0 1 1 3 6 6 8 8 |
| 6 | 3 3 6 6 9 |
| 7 | 2 4 5 5 6 |
| 8 | 0 1 3 5 9 |
| 9 | 2 4 9 |

Key: 6 | 3 means 63

(iii) Median = 66

(iv) Semi-interquartile range = 13.5

(v) P (less than 50 marks) = $\dfrac{3}{30} \times \dfrac{2}{29} = \dfrac{3}{435}$

Question 8

This question tested candidates' ability to:

- draw, interpret and make use of velocity-time graphs;
- calculate and use velocity, acceleration and time in simple equations representing the motion of a particle in a straight line.

Candidates performed best on Parts (a) (i), (b) (i) and (iii) of this question, which required them to draw a velocity-time graph from given information; obtain from its velocity function, $t$-values when a particle is at instantaneous rest and obtaining values for $\dfrac{dv}{dt}$ at particular points in time.

Parts (a) (ii), (b) (ii) and (iv), however, presented the most difficulty to candidates. In Part (a) (ii), candidates were required to find the total distance travelled by the particle from their graph, as well as to find the average velocity of the particle. A few candidates did not use the graph and used instead the formula, distance = speed × time, which gave incorrect values. This formula will only give correct distances if the speed is constant. Other candidates had some problems correctly finding the area under the graph of all parts of the graph. A few candidates did not know how to calculate average velocity, that is, average velocity = $\frac{\text{distance}}{\text{time}}$, with some simply dividing their total distance by 2.

In Part (b) (ii), which required candidates to obtain the distance travelled by the particle between $t = 1$ and $t = 3$ by integrating $v$ to obtain displacement. Whilst a number of candidates correctly obtained that at t = 1, displacement = 7, and at t = 3, displacement = –9, about one-half of them did not know that the distance between t = 1 and t = 3 is 16, as the displacement of –9 meant 9m (or whatever unit of length) to the other side of 0. They experienced difficulty in interpreting distance from the calculated displacement values. In Part (b) (iv), the interpretation of the $\frac{dv}{dt}$ values presented difficulty to about one-half of the candidates, with many, for example, believing that $\frac{dv}{dt} = 0$ implied that the particle was at rest.

**Solutions**: (a) (ii) a) 900 m

b) average velocity = 25.7 m $s^{-1}$

(b) (i) $t = 1$ or t = 5

(ii) distance travelled = 16 m (units)

(iii) a) at $t = 2$, $\frac{dv}{dt} = -6$

b) at $t = 3$, $\frac{dv}{dt} = 0$

(iv) a) at $t = 2$, particle is decelerating

b) at $t = 3$, acceleration is 0, or constant velocity

**Paper 031 – School Based Assessment (SBA)**

Many of the projects submitted were of a high quality and generally related to real-world situations. Many of the submissions received full marks.

The following were general observations on both Projects A and B:

- Some SBA project titles and aims/purpose were not clear, not concise and not well defined. Some titles were too vague, others too wordy.
- In far too many cases, candidates simply re-stated the project's title as its aim/purpose. This often meant that the reason for the project was not stated.
- In a number of cases, the project's aim/purpose did not relate to the given title and sometimes was not linked to any syllabus objective;
- Some submissions did not come with the required CSEC Additional Mathematics rubric;

- Some teachers created their own rubric or made adjustments to the CSEC Additional Mathematics rubric. This is unacceptable, as the project then has to be re-marked using the appropriate CSEC Additional Mathematics rubric;

- A few project submissions appeared to have been downloaded from the Internet;

- There were too many instances of incorrect spelling and grammar. However, candidates used the correct mathematical jargon and symbols appropriately most of the time;

- Some projects were incorrectly categorized and assessed, that is, a Project A being mislabelled and assessed as a Project B, and vice versa. This did create some problems as, for example, Project B requires the collection and analysis of data from an experimental-type activity.

**Specifically for Project A:**

**Mathematical Formulation**

- In stating how the project was going to be done generally most candidates correctly identified all the important elements of the problem and showed understanding of the relationship between the elements.

- The content used in some cases was up only to a Form 3 (or Grade 8) level Mathematics, for example some projects had only simple calculations with area and volume.

**The Problem Solution**

- Assumptions were often not clearly stated. Some candidates, though, did state limitations of the project.

- Explanations were generally not sufficient and also were not placed between every successive step in the problem solution.

- Calculations were often precise but solutions were not clearly stated.

**Application of Solution**

- Many candidates were unaware of how to show that the solution or proof of the given problem was valid.

**Discussion of Findings/Conclusion**

- In some instances the discussion was worthwhile. However, the discussion was not always related to the project's purpose and was often quite broad.

- Conclusions in many instances were not found to be valid. Some candidates stated many conclusions which were not related to the purpose of the project.

- There were only few instances where suggestions for future analyses were stated.

**Specifically for Project B:**

**Method of Data Collection**

- Although candidates indicated what they did, some were not specific enough about the type of data collected and method of sampling.

**Presentation of Data**

- Candidates had the minimum requirement of one table or chart. However, some tables and charts were not properly named or labeled.

- There was not a systematic layout or organization of the tables and charts.

**Mathematical Knowledge (analysis of data)**

- Mathematical concepts in many cases were not used appropriately from the Additional Mathematics syllabus.

- In many cases candidates used concepts and representations up to just Form 2 level Mathematics; for example, some candidates only used pie charts and bar charts in their representation of the data.

- Most of the calculations shown were accurate. However, in some cases no working was seen because some candidates used software to generate the values.

- Generally some analysis was attempted, but the analysis was often not coherent. This could be attributed to the fact that no proper summary table of calculation values was seen, so the markers had to search through the document constantly to link the calculated values with the analysis.

- The candidates were not too clear on two approaches to be used in the analysis. In many cases the two approaches used were of the same concepts; for example, mean, mode and median were presented as different approaches to the analysis, but all of these are measures of central tendency.

**Discussion of Findings/Conclusions**

- In most instances there was no statement of findings.

- Conclusions made were based on the reported findings but often were not related to the purpose of the project. As a result their conclusions were invalid.

- In most cases there were no suggestions for future analysis.

**Recommendations**

The following recommendations hold for continued improvement in this aspect of the Additional Mathematics examinations:

- Teachers are reminded not to use projects taken directly from the specimen papers or from the exemplar projects given in the Additional Mathematics syllabus.

- In the cases where group work is done, candidates are reminded to submit individual reports.

- All projects should have a clear and concise title, and well-defined aim(s) or purpose;

- Where possible the SBA should be related to authentic situations;

- The variables that are being used or measured (Project B) must be clearly stated and described. Variables can be controlled, manipulated and responding;

- The type of sample and sample size, if relevant, must be clearly stated;

- Teachers must ensure that projects which integrate other subject areas utilize concepts as contained in the CSEC Additional Mathematics syllabus;

- If candidates collect their data in a group setting, candidates **must** demonstrate their **individual** effort in relation to analysis (interpretation) and finding(s)/conclusion(s);

- Projects involving dice or playing cards must be more expansive so that candidates can present a more in-depth analysis of the topic under consideration, for example, this could include multiple and non-standard dice or multiple decks of cards;

- As good practice, candidates should be encouraged to cite all sources and insert a reference/bibliography page;

- Teachers should guide students using the assessment criteria found in forms 'Add Math 1 – 5A' and 'Add Math 1 – 5B' which are both available on the CXC website. Teachers can give their students the rubric as a means of guidance in developing their projects;

- Overall, it is clear that training workshops are needed in the area of project work for Additional Mathematics. Therefore, it is recommended that workshops be held throughout the Caribbean. Teachers need to be aware that there are TWO RUBRIC SHEETS 1-5A for project A (the mathematical modelling project) and 1-5B (the statistical project), and to use the appropriate sheet in marking students' projects. It was apparent that some teachers found difficulties in understanding some aspects of the rubric, or were unaware of the existence of the rubric. Teachers need to ensure that the content level in the project coincides with the Additional Mathematics syllabus.

**Paper 03/2 – Alternative to the School Based Assessment (SBA)**

This paper tested candidates' ability to:

- write the equation of a circle; find the points of intersection of a curve with a straight line;

- use the concept of stationary points; differentiate simple polynomials and quotients of simple polynomials.

In Part (a), use of the formula for a circle whose centre was the origin was generally known. Part (b) (i), which required candidates to find an expression for the height of a cuboid given its length, width and volume, as well as Part (b) (ii), which required candidates to write an expression for the total surface area of the cuboid, appeared to be widely known. The formula for total surface area, $2\times(l\times w + l\times h + w\times h)$ was also well known.

In Part (a), candidates had difficulty relating the sides of the square to their equations $x = \pm 5$ and $y = \pm 5$ and hence did not know they could use simultaneous equations to obtain the other coordinates. In Part (b), candidates for the most part did not know that the stationary point occurred when $\frac{dA}{dx} = 0$ and many of them could not differentiate the expression $\frac{2160}{x}$.

Teachers need to emphasize that a stationary point occurs when the gradient function, $\frac{dy}{dx} = 0$. Teachers also need to do more examples on differentiating functions of the form $\frac{a}{x^n}$, where $a$ is a constant and $n$ a positive integer.

**Solutions** : (a) $(5, \sqrt{11})$; $(5, -\sqrt{11})$; $(-5, \sqrt{11})$; $(-5, -\sqrt{11})$;

$(\sqrt{11}), 5)$; $(\sqrt{11}), -5)$; $(-\sqrt{11}), 5)$; $(-\sqrt{11}), -5)$

(b) (i) $h = \frac{360}{x^2}$

Generally, teachers need to ensure that the syllabus is completed with sufficient time to give candidates time to do practice exercise and past papers.

C A R I B B E A N  E X A M I N A T I O N S  C O U N C I L

# REPORT ON CANDIDATES' WORK IN THE

# CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION

**MAY/JUNE 2014**

## ADDITIONAL MATHEMATICS

## GENERAL PROFICIENCY EXAMINATION

## GENERAL COMMENTS

Additional Mathematics was tested for the third time in the May/June 2014 examinations. The subject is intended to bridge a perceived gap between the CSEC General Mathematics and the CAPE Unit l Pure Mathematics.

The intent is to offer CSEC Mathematics students a more seamless transition to the advanced thinking and skills needed for CAPE Unit 1 Mathematics courses, although a student who masters the content and skills set out in the CSEC Mathematics syllabus should be ready to meet the skills and thinking demands of the CAPE Unit 1 Mathematics course.

The examinations consist of four papers:

- Paper 01 – a 45-item multiple choice paper
- Paper 02 – a structured, 'essay-type' paper consisting of eight questions
- Paper 031– a School-Based Assessment (SBA) project component for candidates in schools
- Paper 032 – an alternative to the SBA for out-of-school candidates.

Questions in the examination targeted the content and specific objectives of the Additional Mathematics syllabus (CXC 37/G/SYLL 10) with the aim of ensuring a wide coverage of the syllabus. The questions were designed at the appropriate level to test the following skills: Conceptual Knowledge, Algorithmic Knowledge and Reasoning.

Paper 01 tested content from Sections 1, 2 and 3 of the syllabus and consisted of 45 items. Paper 02 tested content from all four sections of the syllabus. This year the paper consisted of four sections and each section contained two problem-solving questions. The questions in Section 1, 2 and 3 were all compulsory, and the questions in Section 1 were worth 14 marks each, those in Section 2 were worth 12 marks each and those in Section 3, 14 marks each. Section 4 contained two questions, one on Data Representation and Probability and the other on Kinematics and each question was worth 20 marks. Candidates were required to answer only one question from this section. Paper 031 is the SBA component of the examination. Candidates are required to complete one project chosen from two project types; a mathematical modelling project (Project A) and a data handling/statistical analysis project (Project B). The SBA component is worth 20 marks. Private candidates can sit an alternative paper to the SBA, Paper 032, which consists of one in-depth, extended question from Sections 1, 2 and /or 3 of the syllabus. This paper is worth 20 marks.

This year saw a 16 per cent increase in candidates registered for the examinations, up from 3100 candidates in 2013 to 3597 in 2014.

Again this year, a formula sheet was included as part of the question paper on Papers 01, 02 and 03/2.

## DETAILED COMMENTS

### Paper 01 – Multiple Choice

This was a 45 item paper covering Sections 1, 2 and 3 of the syllabus. The mean score on this paper was 39.41, with standard deviation of 12.71 respectively, compared to 35.25 and 13.31 in 2013.

### Paper 02 – Structured Essay Questions

This paper consisted of eight questions, of which Questions 1 to 6 were compulsory. In addition, candidates had to answer either Question 7 or Question 8. The total possible score on this paper was 100 marks. The mean and standard deviation for this paper were 46.32 and 26.01 respectively, compared to 47.82 and 27.27 in 2013.

**Section 1: Algebra and Functions**

Question 1

This question tested candidates' ability to:

- determine whether a given function is one-to-one; find the composite of two functions and determine its domain; determine the inverse of a given function and sketch the given graph and its inverse on the same pair of axes;
- apply the Remainder Theorem to evaluate an unknown coefficient;
- convert a real-world worded problem to obtain a quadratic equation and solve it in order to find the dimensions of a room.

There were 3 462 responses to this question. The mean mark was 7.50 with standard deviation 3.95. Approximately 5 per cent of candidates obtained full marks.

Candidates performed best in Parts (b) and (c) of this question, with many candidates earning full marks for these parts of the question.

Part (b) required candidates to evaluate the unknown coefficient of $x$. Most candidates successfully applied the Remainder Theorem to find this coefficient. Generally candidates were able to make the correct substitution and equate this to the remainder; however some candidates made errors while simplifying the expression.

In Part (c) candidates were required to formulate equations representing the length and width of the room. Most candidates then solved these equations simultaneously to find the required values. However, some candidates used trial and error to arrive at their answers, which at times were correct. Generally many candidates recognised that the positive solution from the quadratic equation was the correct value to be used to find the required length; however some used both positive and negative values to obtain the length without indicating which pair was valid.

Part (a) (i) and (a) (ii) presented the greatest difficulty to candidates.

Part (a) (i) required candidates to show that a given function was NOT one-to-one. Few candidates were able to show that the function was not one-to-one although some simplified the function in a cubic expression correctly using the difference of squares method.

Candidates who obtained full marks were able to do so by either showing that $f\,(–a) = f\,(a)$ for $a \in R$ or making a sketch of the function and applying the horizontal line test. Most candidates, when doing Part (a) (ii) a), were able to correctly find the value of the composite function. However, they were unable to state the domain and when they did, it was done incorrectly.

Part (a) (ii) b) also presented some difficulty to candidates. The inverse of the function was at times incorrectly stated; for example, it was often represented as $2x + 3$. Many candidates also did not even attempt to sketch the two graphs $g(x)$ and their inverse in this part of the question, even though they performed well in other parts of the question.

With respect to candidates' responses on this question, consolidation is needed in the following areas:

- More graph work and analysis of graphs need to be done. Students should be given extensive practice in drawing graphs or, in cases where a sketch is required, in representing the data in an efficient and effective way. They also need to differentiate between a quadratic graph and a linear graph.

- Students must not consider as equivalent, the quotient of a polynomial expression and the remainder of a polynomial expression as it applies to the Remainder Theorem.

- Students should be shown the importance of applying mathematical solutions to real-world situations. For example, they correctly found the length and width of the room but did not state these values to represent such. This occurrence had little impact here since once the correct values were seen they were credited. However, if these values were to be used for further analysis of the question some difficulty might have arisen.

**Solutions are**: (a) (i) $f(-a) = f(a)$ thus $f$ is many- to- one

(ii) a) $-\frac{1}{4}x^2 + 3x - 8 \quad x \in \Re$

b) $g^{-1}(x) = 2(x+3) \quad x \in \Re$

(b) $a = 3$

(c) width $x = 6$ cm and length $y = 18$ cm

<u>Question 2</u>

This question tested candidates' ability to:

- express a quadratic function *f(x)* in the form $k + a(x+h)^2$ where *a, h* and *k* are constants; determine the function's maximum value and the value of *x* for which the maximum occurred;
- find the solution set of a quadratic inequality;
- identify a geometric series and calculate the sum to infinity of the geometric series.

There were 3 522 responses to this question. The mean mark was 8.19 with standard deviation 4.27.

Candidates performed best in Part (c) of this question. However approximately 20 per cent of the candidates incorrectly used the common ratio as 10. To show that the series was geometric approximately 25 per cent of the candidates used the first two terms only. A few even treated the series as an arithmetic progression and obtained a common difference of 0.18 between the first two terms.

Parts (a) and (b) were equally challenging to the candidates. In Part (a) (i) candidates demonstrated difficulty in factorising from the given function and in completing the square correctly. Most of the candidates used the alternative approach in which they recalled expressions for the required *h* and *k* based on their previous knowledge. Some candidates made errors in recalling these expressions and the majority of candidates who were able to present a correct expression made errors in computing the numerical values of *h* and *k*. In Part (a) (ii) most candidates were able to deduce their correct maximum value however some candidates provided a minimum value of *f*(*x*) although the question was clearly stated. Some candidates provided the coordinates of the maximum point instead of providing the function's maximum value. In Part (a) (iii) some candidates did not see the connection of this question to their response in Part (a) (i).

Some of the candidates could not obtain the correct set of values of *x* although they correctly factorised and determined the correct critical values. A few candidates failed to change the sign of the inequality when they multiplied by −1. For the most part, however, the attempt of factorisation was well done.

This question was generally well done by the majority of candidates.

**Solutions are**: (a) (i) $9 - 2(x+3)^2$

(ii) Maximum value $f(x) = 9$

(iii) Value of $x$ at maximum $= -3$

(b) Solution set: $\{x: x \leq \frac{-1}{2}\} \cup \{x: x \geq 3\}$

(c) (i) common ratio of $\frac{1}{10}$

(ii) $\frac{2}{9}$

**Section 2: Coordinate Geometry, Vectors and Trigonometry**

Question 3

This question tested candidates' ability to:

- apply the perpendicular property for two lines; determine the equation of a circle, given its radius, and whose centre is the point of intersection of two given lines;
- apply properties of perpendicular vectors and determine the magnitude of a displacement vector.

Candidates demonstrated good recall of knowledge of Coordinate Geometry and vectors.

In Part (a) (i) candidates correctly expressed the condition for two straight lines perpendicular to each other as $m_1 \times m_2 = -1$, $m_2 = -1/m_1$ or the equivalent in words.

In Part (a) (ii) candidates correctly recalled the equation of the circle in the form $(x - a)^2 + (y - b)^2 = r^2$ where $(a, b)$ are the coordinates of the centre of the circle and $r$ is the radius. This form of the circle equation was most suitable for the solution required. Some candidates correctly recalled the expanded form $x^2 + y^2 - 2fx - 2gy + c = 0$.

In Part (b) (i), candidates recognised that the dot product $\overrightarrow{RT} \bullet \overrightarrow{RS} = 0$ was the condition required for proving that $<TRS = 90^0$. Other conditions presented in candidates' responses were for angle $RTS = 90^0$, cos $<RTS = \frac{\overrightarrow{RT}.\overrightarrow{RS}}{|\overrightarrow{RT}.||\overrightarrow{RS}|} = 0$, requiring knowledge of the length of a vector.

- The product gradient RT × gradient RS = –1
- Graphical method
- An unknown method involving ratios

In Part (b) (ii) candidates correctly recalled Pythagoras' Theorem to determine the length of the hypotenuse. Candidates faced challenges throughout their responses in terms of correctly applying the content to data.

In Part (a) (i) candidates were unable to manipulate the two equations, given in the form $-x + 3y = 6$ and $kx + 2y = 12$, to obtain the gradients. Only a few candidates correctly identified $-k/2$ as one of the gradients. As a consequence candidates were unable to formulate the equation $-1/3 \times -k/2 = -1$, resulting in incorrect values for $k$.

The layout of both the given equations, $x + 3y = 6$ and $kx + 2y = 12$ prompted candidates to obtain the value of $k$ by solving the equations simultaneously, not recognising that two equations containing three unknowns cannot be solved. This attempt by candidates was abandoned at some point in the procedure.

The follow through allowed in Part (a) (ii) for the incorrect value of $k$ made it possible for candidates to be awarded both marks for solving the simultaneous equations to obtain the coordinates of the centre of the circle. Some candidates wrote the equation of the circle incorrectly due to the non-reversal of signs when substituting the coordinates of the centre of the circle in the correct equation of the circle.

The use of the word "show" in the stem of Part (b) (i) led to the use of a variety of strategies in the proof.

In both Parts (b) (i) and (b) (ii) incorrect selection of vectors for substitution in the dot product or the formula for the length of a straight line was frequently seen. Position vectors were commonly seen rather than the correct translation vectors. Incorrect terminology in writing vectors was also seen e.g. $\overrightarrow{RT} \bullet \overrightarrow{RS}$ was written $R \times S$).

Consolidation in Coordinate Geometry is required in the area of expressing equations of straight lines given the form $ax + by = c$ to the form $y = mx + c$ in order to better facilitate the identification of the gradient. Competency in developing proofs using a vector method also needs to be emphasized.

**Solutions are**: (a) (i) $k = –6$

(ii) Equation of the circle $(x + \frac{6}{5})^2 + (y - \frac{12}{5})^2 = 5^2$

(b) (i) RT.RS = 0 for angle TRS $= 90^0$

(ii) $\left|\overline{ST}\right| = \sqrt{26}$

Question 4

This question tested candidates' ability to:

- determine the area of a sector of a circle and find the area of a shaded region using the area of the sector and that of a triangle;
- use the compound angle formula for $\cos(x + \frac{\pi}{6})$
- prove a trigonometric identity.

Candidates performed best on Part (b), which required proving the compound angle formula. Candidates were able to apply the formula given in the formula sheet and correctly substitute the values given. Candidates were also successful in factorizing the $\frac{1}{2}$ to give the result. Some candidates, though, used both sides of the proof to attain the answer.

Teachers are urged to show the students the method of substituting correctly and how to use either the left hand side or the right hand side of the equation to achieve the proof.

Part (a) presented the most difficulty for candidates. A number of candidates incorrectly calculated the area of the triangle and subsequently calculated an incorrect area of the shaded region, H. However, most candidates knew that the area of the shaded region, *H*, was calculated by subtracting the area of the sector AOB from the area of the triangle OAC. Most candidates also attained full marks for Part (i) in finding the area of the sector. Teachers are encouraged to have students understand the steps needed to complete questions of this nature, that is, the correct use of the formula Area $= \frac{1}{2} ab\sin C$.

In Part (c), most candidates received marks for substituting correctly for $\tan\theta = \frac{sin\theta}{cos\theta}$ and $\sin^2\theta = 1 – \cos^2\theta$. However, simplification after the substitution, to get the required result $1 + \frac{1}{cos\theta}$, proved to be most difficult. Candidates did not recognize the difference of squares $1 – \cos^2\theta = (1 + \cos\theta)(1 - \cos\theta)$ and, as a result, lost marks at that part of the proof.

Teachers are encouraged to reinforce the use of the difference of squares examples including $1 - \cos^2\theta$ and $1 - \sin^2\theta$.

**Solutions are**: (a) (i) Area of sector = 28.35 $cm^2$

(ii) Area of shaded region = 5.76 $cm^2$

**Section 3: Introductory Calculus**

Question 5

This question tested candidates' ability to:

- determine the equations of the tangent to a curve;
- use the concept of stationary points; locate the maximum and minimum stationary points, by considering the sign changes of the derivative; calculate the second derivative of polynomials; and use the sign of the second derivative to determine the nature of stationary points.

Candidates' demonstration of the understanding of the Calculus and Coordinate Geometry content in this section of the syllabus was commendable. Candidates recognised that in Part (a)

- the first derivative would give rise to an expression for the gradient of the tangent, $-2x + 4$.
- substituting the $x$-coordinate of the given point in $-2x + 4$ would give the value of gradient of the tangent.
- the general linear equation $y = mx + c$ or $y - y_1 = m(x - x_1)$ would give rise to the equation of the tangent of the curve.

For Part (b) (ii)

- differentiating the function $f(x) = 2x^3 - 9x^2 - 24x + 7$ to obtain the first derivative $f'(x) = 6x^2 - 18x - 24$ and equating the resulting quadratic equation to zero would lead to solutions which are the stationary points of the curve.
- further differentiating the first derivative to obtain the second derivative $f''(x) = 12x - 18$ would enable them to determine the nature of EACH of the stationary points.

In Part (a) the candidates showed competence in correctly obtaining the first derivative $dy/dx = \mathbf{4 - 2x}$ to represent the gradient of the given quadratic curve $y = 3 + 4x - 3x^2$ and substituting $x = 3$ from the given point to obtain $m = -2$.

In Part (b) (i) candidates solved the quadratic equation $6x^2 - 18x - 24 = 0$ using the methods of the factorisation, completing the square and use of the quadratic formula.

In Part (b) (ii) candidates were able to obtain the second derivative $f''(x) = 12x - 18$. The follow through $x$ values from Part (b) (i) was an added advantage to candidates who were able to substitute in $f''(x) = 12x - 18$ to differentiate between the maximum and minimum stationary points of the function. Many candidates obtained full marks in this part of the question.

Other strategies used in candidates' responses to differentiate between the maximum and minimum stationary points were as follows:

- Sketching the graph of $f(x)$
- Examining the coefficient of $x^3$ to determine the shape of the graph and the direction of the stationary points.

Areas of ambiguity in Part (a) were

- whether the equation of the tangent to the curve or the normal was required. Some candidates converted the gradient m = −2 to m = $\frac{1}{2}$ in order to substitute into general equation thus finding the equation of the normal to the curve at P (3, 6) instead of the equation of the tangent.

- the use of the given point on the curve P (3, 6). Some candidates used the gradient expression to formulate the equation $-2x + 4 = 0$ to find a new value of $x = 2$ for substituting in the gradient expression. Others used the given function to formulate a quadratic equation in order to find $x$ values for substituting in the gradient expression.

- multiplying through the curve $y = 3 + 4x - x^2$ by –1 before differentiating. This gave rise to the incorrect gradient expression $2x + 4$.

Weakness in basic algebraic manipulations was evident in this part of the question, including

- transposition of terms in an equation
- factorisation of quadratic expressions.

Transposing the equation of the tangent obtained to the required format posed a challenge to candidates who often expressed their equation in the form $y = mx + c$. Attempts at transposing the equation showed incorrect signs associated with the coefficients of $x$ and $y$.

In Part (b) (i), factorising the quadratic equation $6x^2 - 18x - 24 = 0$ was a challenge to many candidates. Incorrect factors were obtained. Use of the quadratic formula and completion of the square method were often unsuccessful and abandoned before arriving at solutions to the equation.

Some candidates who obtained the correct x-coordinates did not proceed to calculate the corresponding $y$–coordinates to obtain ALL the stationary points.

With respect to candidates' responses in this question, there was good demonstration of understanding of the content areas throughout the question. Deficiencies in algebraic manipulations for weaker candidates can be a drawback to these candidates' performance. These deficiencies can be eliminated by consistent and appropriate worksheets with specific focus on developing skills in algebraic manipulations with accompanying effective feedback to candidates.

**Solutions are:** (a) Equation of the tangent to the curve at (3, 6) is $2x + y - 12 = 0$.

(b) (i) Stationary points are (–1, 20) and (4, –105)

(ii) (–1, 20) maximum point, (4, –105) minimum

Question 6

This question tested candidates' ability to:

- evaluate a definite integral;
- compute a definite trigonometric integral leaving the answer in surd;
- formulate the equation of a curve given its gradient function and points on the curve; find the area of a finite region in the first quadrant bounded by a curve and the $x$-axis and the lines $x = 3$ and $x = 4$.

Candidates performed best on Part (b), except for those candidates that did not know the surd form for $\cos\frac{\pi}{3}\, and \sin\frac{\pi}{3}$. Part (c) (i) was fairly well done; however, there were many candidates who were unable to correctly calculate the value of the constant of integration, $c$, despite the fact that they were able to correctly integrate the function and substituted the point (2, –5). In Part (c) (i), some candidates could not obtain the correct equation of the curve. Even among those who knew to integrate the gradient function to obtain this equation, some did not include a constant of integration and hence obtained an incorrect equation.

Part (a) and Part (c) (ii) presented the most difficulty to candidates. In Part (a), many candidates used the incorrect formula. “the integral of a product is the product of the integrals”. Others did not integrate at all, but simply substituted the limits in the function.

In Part (c) (ii), generally candidates knew that area is the integral of a function between limits. However, many used the function as $y = 6x^2 - 1$, which was actually the given gradient function, $\frac{dy}{dx} = 6x^2 - 1$, instead of the equation of the curve obtained in Part (c) (i).

**Solutions are**: (a) 48

(b) $2\sqrt{3} + 1$

(c) (i) $2x^3 - x - 19$ Area = 65 units$^2$

**Section 4: Basic Mathematical Applications**

Question 7

This question tested candidates’ ability to:

- use the formula $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ to calculate probabilities;
- construct and use possibility space diagrams to solve problems involving probability;
- use the given stem and leaf diagram to determine quartiles and the median and to construct a box- and-whisker plot.

Candidates performed best on Part (b) (i), which required them to complete the possibility space diagram, and they also able to interpret the correct sample space of 36. For Part (a), most candidates used previous knowledge of sets and Venn diagrams to evaluate the correct probabilities.

Candidates performed well on the interpretation of the stem and leaf diagram to calculate the median and quartiles for the data given, recognizing that the median was the 26$^{th}$ observation, 71. The majority of the candidates also knew to construct a box-and-whisker plot to illustrate the data given. However, the comment on the skewness of the distribution proved to be very difficult. Candidates opted to omit the answer of negatively skewed and some did not provide a mathematical comment on the plot.

Teachers are advised to stress on the skewness of a box-and-whisker plot to describe the distribution of the data. The box-and-whisker plot should be accurately drawn on graph paper as points were deducted for incorrect scale.

Part (b) (iii) was also difficult in that most candidates who attempted to calculate the difference between the two dice forgot to multiply by 2 since the answers could be reversed. Very few candidates used a possibility space diagram to calculate the probabilities, but instead listed the possible combinations.

Generally, candidates displayed knowledge on the Statistics option but made some simple mistakes. For example, candidates need to know that probabilities are between 0 and 1 inclusive and that any answer above 1 is incorrect.

Candidates performed best on Part (a) (i) of this question, which required them to complete the construction of a tree diagram. Candidates knew that the sum of each branch equated to 1. However, having drawn the tree diagram, many candidates did not use it to answer Parts (a) (ii) and (iii). Part (a) (ii), on the conditional probability, was badly done, and it appeared that most candidates did not even know that the question required the use of conditional probability. In Part (a) (iii), most candidates were able to get the probability for Event V, and also to say which was the more likely event.

Parts (b) (i), (ii), (iv) and (v) presented the most difficulty to candidates. In Part (b) (i), which asked candidates to state an advantage of a stem and leaf diagram versus a box and whiskers plot to display the given data, a number of candidates stated their personal preference rather than an advantage, for example, 'the stem and leaf is easier to draw and understand'. In Part (b) (ii), which required candidates to construct the actual stem and leaf diagram, a number of candidates drew actual tree diagrams. For candidates who did draw a stem and leaf diagram, a number of them lost a mark for not including a key to the diagram. In Part (b) (iv) candidates did not know the formula for semi-interquartile range, or did not know how to work this out. In many cases they could not correctly find the lower and upper quartiles, and many did not know to divide this difference by two. In Part (b) (v), some candidates mistook the sample space for the marks. Many other candidates did not recognize the sample space being reduced, and so could not correctly give the probability for the second student scoring less than 50 marks on the exam.

**Solutions are**: (a) (i) $\frac{3}{20}$

(ii) $\frac{11}{60}$

(b) (i)

| | | | | | | |
|---|---|---|---|---|---|---|
| **6** | 7 | 8 | 9 | 10 | 11 | 12 |
| **5** | 6 | 7 | 8 | 9 | 10 | 11 |
| **4** | 5 | 6 | 7 | 8 | 9 | 10 |
| **3** | 4 | 5 | 6 | 7 | 8 | 9 |
| **2** | 3 | 4 | 5 | 6 | 7 | 8 |
| **1** | 2 | 3 | 4 | 5 | 6 | 7 |
| | **1** | **2** | **3** | **4** | **5** | **6** |

(ii) a) $P(S > 9) = \frac{1}{6}$

b) $P(S \leq 4) = \frac{1}{6}$

(iii) $a = \frac{5}{18}$ $b = \frac{1}{6}$ $c = \frac{1}{18}$

(c) (i) median = 71 lower quartile = 56 upper quartile = 79

(ii) Draw box-and-whisker plot
Shape is negatively skewed

**Question 8**

This question tested candidates' ability to:

- interpret and make use of velocity-time graphs to find distance , deceleration and time;
- apply the appropriate rate of change such as $a = \frac{dv}{dt}$ and use such to calculate time when velocity is maximum, maximum velocity and distance moved by particle from rest to where the particle attains maximum velocity.

Candidates performed best on Parts (a) (i) and (iii) and (b) (i) and (ii) of this question which required them to calculate the distance from a velocity-time graph, the time it took to decelerate to $10ms^{-2}$, obtain the acceleration when the velocity is maximum and calculate the maximum velocity.

Parts (a) (ii), (iv) and (b) (iii), however, presented the most difficulty to candidates. In Part (a) (ii), candidates were required to find the time travelled by the particle when it stated to decelerate, as well as to find the average acceleration of the particle. Most candidates looked at the graph and estimated the time as 10 seconds instead of using the formula distance = total area under the curve to calculate the correct time. A few candidates were able to correctly estimate that the car took 8 seconds to decelerate and then found the average velocity using average velocity = $\frac{\text{dis tan ce}}{\text{time}}$.

In Part (a) (iv), many candidates were unable to correctly find the additional time after 90 seconds for the particle to come to its final rest. Candidates did not use the area of the triangle to calculate the time, but rather some divided the 30 metres it further travelled by 10 $ms^{-1}$.

Part (b) (iii), required candidates to calculate the distance moved by the particle from rest to where the particle attained maximum velocity, that is they were expected to evaluate $\int_0^5 0.72t^2 - 0.096t^3 dt$ .

Some candidates used the first function with limits between 5 and 10. Others used the second function with limits between 0 and 5. Still others integrated the first function with limits between 0 and "his" time that represented where the particle reached maximum velocity.

**Solutions are**:

(a)
- (i) 750 m
- (ii) deceleration = 1.875 m $s^{-2}$
- (iii) 52 seconds
- (iv) 15 m $s^{-1}$

(b)
- (i) $t$ = 5 seconds
- (ii) max velocity = 6 m $s^{-1}$
- (iii) distance travelled = 15 m

**Paper 03/2-Alternate to the School Based Assessment (SBA)**

This paper tested candidate ability to:

- construct an geometric series from circumscribed rectangles and obtain the sum of its first *n* terms.
- apply logarithms to a problem involving the transformation of a given relationship to a linear form and use its graph to unknown constants.

For Part (a), candidates were required to use the method of circumscribed rectangles to:

- determine the width of each of n subdivisions
- determine the point of each subdivision
- determine the height of each circumscribed rectangle
- determine area of each circumscribed rectangle
- determine the area of the circumscribed rectangles
- compute the area of the circumscribed rectangles as the progression increases
- determine the limit to which these values approaches

For Part (b), given a logarithm function:

- convert to the linear form
- find the values a and n by finding gradient and y intercept

There were approximately 120 responses to this paper. No candidate received the full 20 marks. The highest mark attained was 17. The mean score was 8.69 with a standard deviation of 7.75

Candidates, in general, were unable to do Part (a) of the question. About 50 % substituted into $S_n$ to get values for a but were unable to identify that the limit approached 0.5.

For Part (b) most candidates were successful in finding the logarithms of the table and plotting the graph. The scale did not accommodate all the points to be plotted. Therefore, some of the candidates were unable to make the necessary adjustments. They were however able to convert the equation to the logarithm format, as well as to calculate the values of *a* and *n* from the graph. In general, the question was poorly done.

**Solutions are**: (a) (i) $\frac{1}{n}$ .

(ii) $x_1 = \frac{1}{n}$ , $x_2 = \frac{2}{n}$, $x_3 = \frac{3}{n}$ ….. . $x_{n-1} = \frac{n-1}{n}$ .

(iii) $h_1 = \frac{1}{n}$, $h_2 = \frac{2}{n}$, $h_3 = \frac{3}{n}$,…….. $h_n = 1$

(iv) $A_1 = \frac{1}{n^2}$, $A_2 = \frac{2}{n^2}$, $A_3 = \frac{3}{n^2}$ ….. $A_n = \frac{1}{n}$ .

(vi) 0.55 0.525 0.51 0.505

(b) Approaches 0.5 as n gets larger

$\log y = \log a + n \log x$
*n* is approximately 4
*a* is approximately 3 .

## Paper 031 – School-Based Assessment (SBA)

Many of the projects submitted were of a high quality and generally related to real-world situations. The mean mark awarded on the SBA was 33.06 (out of a possible score of 40 marks), with standard deviation of 6.30.

A number of observed weak areas related to sample submissions, related to the submissions themselves as well as to the marking of these samples. The following were general observations on both Project A and Project B:

- Some SBA project titles and aim(s) or purpose were not clear, not concise and not well defined. Some titles were too vague, others too wordy.
- In far too many cases, students simply re-stated the project's title as its aim or purpose. This often meant that the reason for the project was not stated.
- In a number of cases, the project's aim or purpose did not relate to the given title and sometimes was not linked to any syllabus objective;
- Some submissions did not come with the required CSEC Additional Mathematics rubric;
- Some teachers created their own rubric or made adjustments to the CSEC Additional Mathematics rubric. This is unacceptable, as the project then has to be re-marked using the appropriate CSEC Additional Mathematics rubric;
- A few project submissions appeared to have been downloaded from the Internet;
- There were few instances of incorrect spelling and grammar. However, candidates used the correct mathematical jargon and symbols appropriately most of the time;
- Some projects were incorrectly categorized and assessed, that is, a Project A being labelled an assessed as a Project B, and vice versa. This did create some problems as, for example, Project B requires the collection and analysis of data from an experimental-type activity.

### Comments specifically for Project A:

### Mathematical Formulation

- In stating how the project was going to be done most students correctly identified all the important elements of the problem and showed understanding of the relationship between the elements.

### The Problem Solution

- Assumptions were often not clearly stated. Some students, though, did state limitations of the project.
- Explanations were generally not sufficient and also were not placed between every successive step in the problem solution.
- Calculations were often precise but solutions were not clearly stated.

### Application of Solution

- Many students were unaware of how to show that the solution or proof of the given problem was valid; was necessary to see how the candidate's solution could be applied elsewhere or to be able to substitute their values to check for validity.

### Discussion of Findings/Conclusion

- In some instances the discussion was worthwhile. However, the discussion was not always related to the project's purpose and was often quite broad.
- Conclusions in many instances were not found to be valid. Some candidates stated many conclusions which were not related to the purpose of the project.
- There were few instances where suggestions for analyses were not stated or included in the project.

**Comments specifically for Project B:**

**Method of Data Collection**

- Although students indicated what they did, some were not specific enough about the type of data collected and method of sampling. Since the marking rubric was not specific as to what is needed, the candidates were not penalized.

**Presentation of Data**

- Too many students had the minimum required one table or chart; however, some tables and charts were not properly named or labelled, for example, graphs produced using software programs.
- There was not a systemic layout or organization of the tables and charts

**Mathematical Knowledge (Analysis of Data)**

- In this section, the student was expected to outline <u>HOW</u> the project was going to be done.
- Mathematical concepts in many cases were not used appropriately from the Additional Mathematics syllabus. Many projects utilized content at Form 3 level or below.
- Most of the calculations shown were accurate. However, in some cases no working was seen because some students used software to generate the values.
- Generally some analysis was attempted, but the analysis was often not coherent. This could be attributed to the fact that no proper summary table of calculation values was seen, so the markers had to search through the document constantly to link the calculated values with the analysis.
- The students were often not very clear on which two approaches were to be used in the analysis. In many cases the two approaches used were of the same concepts, for example, mean, mode and median were presented as different approaches to the analysis, but all of these are measures of central tendency.

**Discussion of Findings/Conclusions**

- In most instances there was no statement of findings, in some cases if it was in the project, it was not properly placed.
- Conclusions made were based on the reported findings but often were not related to the purpose of the project. As a result this made their conclusions invalid.
- In few cases there were no suggestions for future analysis. This was an improvement for last year 2013.

**<u>Plagiarism</u>**

Plagiarism continued to be a problem in the Additional Mathematics projects bring submitted for moderation. There were cases where projects were taken directly from the specimen papers as well from the exemplar projects given in the Additional Mathematics syllabus. This was noted among last year's submissions, and the practice continued in this year's submissions. The concern here is that the teachers associated appeared to be accepting these as legitimate projects, when it is clear that they are not the candidates' own work. This is must be stopped.

There were other instances where candidates had obviously done the project as a group. However, they submitted the same data, same mathematical formulations/solution, same diagrams, graphs and tables, same discussions and same conclusion, in some cases even printing and submitting identical copies. This practice must be stopped. Even where the project is done as a group, ALL aspects of the write-up must show candidates' individual effort. Only the raw data collected must be the same.

The following recommendations hold for continued improvement in this aspect of the Additional Mathematics examinations:

- All projects should have a clear and concise title, and well-defined aim(s) or purpose;
- Where possible the SBA should be relevant to authentic situations;

- The variables that are being used (Project B) must be clearly stated and described. Variable can be controlled, manipulated and responding;
- The type of sample and sample size, if relevant, must be clearly stated;
- Teachers must ensure that projects which integrate other subject areas utilize concepts as contained in the CSEC Additional Mathematics syllabus;
- If students collect their data in a group setting, they MUST demonstrate their INDIVIDUAL effort in relation to analysis (interpretation) and finding(s)/conclusion(s);
- Projects involving dice or playing cards must be more expansive so that candidates can present a more in-depth analysis of the topic under consideration;
- As good practice, students should be encouraged to cite all sources and insert a reference/bibliography page;
- Teachers should guide students using the assessment criteria found in forms ‘Add Math 1- 5A’ and ‘Add Math 1-5B’ which are both available on the CXC website. Teachers can give their students the rubric as a means of guidance in developing their projects.

It is clear that training workshops are needed in the area of project work for Additional Mathematics. Therefore, it is recommended that workshops be held throughout the Caribbean. Teachers need to be aware that there are TWO RUBRIC SHEETS 1-5A for Project A (the mathematical modelling project) and 1-5B for Project B (the statistical project), and to use the appropriate sheet in marking students’ projects. It was apparent that some teachers found difficulties in understanding some aspects of the rubric, or were unaware of the existence of the rubric. Teachers need to ensure that the content level in the project coincides with the Additional Mathematics syllabus.

**Paper 032 – Alternative to School-Based Assessment (SBA)**

This paper tested candidates’ ability to:

- construct a geometric series from circumscribed rectangles and obtain the sum of its first *n* terms.
- apply logarithms to a problem involving the transformation of a given relationship to a linear form and use its graph to unknown constants.

For Part (a), candidates were required to use the method of circumscribed rectangles to:

- determine the width of each of n subdivisions
- determine the point of each subdivision
- determine the height of each circumscribed rectangle
- determine the area of each circumscribed rectangle
- determine the area of the circumscribed rectangles
- compute the area of the circumscribed rectangles as the progression increases
- determine the limit to which these values approaches

For Part (b), given a logarithm function:

- convert to the linear form
- find the values *a* and *n* by finding gradient and y intercept

There were approximately 120 responses to this paper. No candidate received the full 20 marks. The highest mark attained was 17. The mean score was 8.69 with a standard deviation of 7.75

Candidates, in general, were unable to do Part (a) of the question. About 50 per cent substituted into $S_n$ to get values for *a* but were unable to identify that the limit approached 0.5.

For Part (b) most candidates were successful in finding the logarithms of the table and plotting the graph. The scale did not accommodate all the points to be plotted. Therefore, some of the candidates were unable to make the necessary adjustments. They were however able to convert the equation to the logarithm format, as well as to calculate the values of *a* and *n* from the graph. In general, the question was poorly done.

**Solutions are**: (a) (i) $\frac{1}{n}$ .

(ii) $x_1 = \frac{1}{n}$ , $x_2 = \frac{2}{n}$ , $x_3 = \frac{3}{n}$ ….. . $x_{n-1} = \frac{n-1}{n}$ .

(iii) $h_1 = \frac{1}{n}$ , $h_2 = \frac{2}{n}$ , $h_3 = \frac{3}{n}$ ,…….. $h_n = 1$

(iv) $A_1 = \frac{1}{n^2}$ , $A_2 = \frac{2}{n^2}$ , $A_3 = \frac{3}{n^2}$ ….. $A_n = \frac{1}{n}$ .

(vi) 0.55 0.525 0.51 0.505

(b) Approaches 0.5 as n gets larger

log *y* = log *a* +*n* log*x*
*n* is approximately 4
*a* is approximately 3 .

C A R I B B E A N  E X A M I N A T I O N S  C O U N C I L

# REPORT ON CANDIDATES' WORK IN THE CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION

**MAY/JUNE 2015**

**ADDITIONAL MATHEMATICS**
**GENERAL PROFICIENCY EXAMINATION**

## GENERAL COMMENTS

Additional Mathematics was tested for the fourth time in the May/June 2015 CSEC examinations. The subject is intended to bridge a gap between CSEC General Proficiency Mathematics and the CAPE Unit 1 Pure Mathematics.

The intention is to offer candidates progressing form the CSEC level in Mathematics to the CAPE Unit 1 level in Pure Mathematics/Applied Mathematics a seamless transition to the advanced thinking required for these courses, although a candidate who masters the content set out in the CSEC syllabus for General Proficiency Mathematics should be armed with the tools needed to be successful at the CAPE proficiency level.

As established, the examination consists of three papers:

- Paper 01 – a 45-item multiple choice paper
- Paper 02 – a structured, 'essay-type' paper consisting of eight questions
- Paper 031 – a School-Based Assessment (SBA) project component for candidates in approved educational institutions
- Paper 032 – an alternative to SBA for out-of-school (private) candidates

Questions in the examination targeted the content and specific objectives of the Additional Mathematics syllabus (CXC 37/G/SYLL 10) with the aim of ensuring a wide coverage of the syllabus. The questions were designed at the appropriate level to test the following skills: Conceptual Knowledge, Algorithmic Knowledge and Reasoning.

Paper 01 tests content from Sections 1, 2 and 3 of the syllabus and consists of 45 items. Paper 02 tests content from all four sections of the syllabus. This year the paper consisted of four sections and each section contained two problem-solving questions. The questions in Section 1, 2 and 3 were all compulsory and each question in these sections were worth 14, 12 and 14 marks respectively. Section 4 contained two questions, one on Data Representation and Probability and the other on Kinematics and each question was worth 20 marks. Candidates were required to answer only one question from this section. Paper 031 is the SBA component of the examination. Candidates are required to do one project chosen from two project types: a mathematical modelling project (Project A) and a data handling/statistical analysis project (Project B). The SBA component is worth 20 marks. Private candidates can sit an alternate paper to the SBA, Paper 032, which consists of one in-depth, extended question from Sections 1, 2 and/or 3 of the syllabus. This paper is worth 20 marks.

The percentage of candidates earning Grades I–III was 73 per cent, compared with 77 per cent in 2014.

This year saw an 11.6 per cent increase in candidates registered for the examination, up from 3609 candidates in 2014 to 4016 in 2015.

Again this year, a formula sheet was included as part of the question paper on Papers 01, 02 and 032.

## DETAILED COMMENTS

### Paper 01 – Multiple Choice

This was a 45 item paper covering Sections 1, 2 and 3 of the syllabus. The mean score on this paper was 39.54 with a standard deviation of 13.36 respectively, compared with 39.41 and 12.71 in 2014.

### Paper 02 – Structured Essay Questions

This paper consisted of eight questions, of which Questions 1 to 6 were compulsory. Candidates had to choose one of Questions 7 and 8. The total possible score on this paper was 100 marks. The mean and standard deviation for this paper were 47.42 and 26.98 respectively, compared with 46.32 and 27.27 in 2014.

**Section 1: Algebra and Functions**

Question 1

This question tested candidates' ability to:

- Determine the value of $g(f(2))$ given the two functions $g$ and $f$.
- Determine the inverse of a function given in the form $\frac{m(x)}{n(x)}$, where $m(x)$ and $n(x)$ are linear functions.
- Completely factorize a cubic equation given one of its linear factors.
- Use the laws of indices to solve an indicial equation.
  Use the laws of logarithms to solve a logarithmic equation.

There were about 4016 responses to this question. The mean mark was 8.41 with a standard deviation of 4.19.

Candidates performed best on Parts (a) and (b), as they should at this level. Candidates who did poorly in these parts generally performed poorly throughout the entire question.

In Part (a), most candidates were able to achieve full marks. Some candidates attempted to substitute $f(x)$ into $g(x)$ and made mistakes while doing the algebra. Few candidates were seen calculating $f(2)$ first and then substituting this value into $g(x)$ which resulted in fewer mistakes.

**Common Errors**

Simple arithmetic miscalculations were made, for example, 20 – 3 was given as '−17' instead of *17*.

The majority of candidates performed very well on Part (b). Even weaker candidates were able to achieve a mark by interchanging $x$ and $y$.

**Common Errors**

Candidates misread the original function, which led to incorrect factorization of $y$.

Candidates 'forgot' to interchange $x$ and $y$.

Part (c) was done relatively well. The majority of candidates recognized the application of the factor theorem and correctly used the long division method to obtain a quadratic equation, which was then factorized. Some used a comparison of coefficients approach and a trial and error approach while a few used the synthetic division method in answering this question. The candidates who were most successful used the long division method.

**Common Errors**

Candidates did not factorize their quadratic quotient to obtain the other two linear factors.

Many proved ($x$ – 2) was a factor of $k(x)$, but this was given in the question.

It appears as though some candidates used programmable calculators to find the roots of the cubic equation and were therefore unable to provide the necessary supporting statements. Simple algebraic/arithmetic errors were seen in the long division method.

In Part (d) (i), candidates recognized the need to change each side to a common base, but many of them were unable to correctly apply laws of indices and solve the resulting simple linear equation presented by the equating of indices. Some candidates applied logarithms to both sides, but few were successful as they applied the laws of logarithms incorrectly.

**Common Errors**

Equating indices of different bases
Expanding brackets incorrectly when simplifying the indices
Inability to recognize ¼ = $4^{-1}$ (negative indices)

Part (d) (ii) presented the most difficulty to candidates. Candidates did not demonstrate a sound knowledge in logarithms. Most showed confusion when ‘dropping logs’, as they simply removed the log function and proceeded to solve the question as is.

Some candidates who correctly applied the laws of logarithms, transposed incorrectly resulting in an incorrect quadratic equation while those who obtained the correct quadratic equation lost a mark as a result of their inability to factorize correctly.

**Common Errors**

$\log_a M + \log_a N = \log_a (M + N)$ instead of $\log_a M + \log_a N = \log_a (MN)$
$3^0 = 0$ *instead of* $3^0 = 1$

**Factorization Errors**

With respect to candidates’ responses on this question, it is recommended that candidates be given more practice questions. This will allow them to understand and apply appropriate laws where and when needed.

**Solutions**

(a) $gf(2) = 33$

(b) $h^{-1}(x) = \dfrac{2x+5}{x-3} \quad x \in \mathbf{R}, x \neq 3$

(c) $2x^3 - 5x^2 + x + 2 = (x-2)(\text{x}-1)(2x+1)$

(d) (i) $x$ = - 2.5 (ii) $x$ = 2 and $x$ = 3

Question 2

This question tested candidates' ability to:

- Express a quadratic function *f(x)* in the form $a(x+b)^2+c$ where *a, b* and *c* are constant
- Determine the coordinates of the function's minimum point.
- Use the sum and product of the roots of a quadratic equation to determine the value of $\frac{1}{\alpha}+\frac{1}{\beta}$, where $\alpha$ and $\beta$ are the roots of the equation.
- Determine, algebraically, the point of intersection of a non-linear curve and a straight line.
- Use the formula for the $n^{th}$ term of an arithmetic progression to calculate a particular term given the first term and the common difference.

There were 3522 responses to this question. The mean mark was 8.79 with a standard deviation of 4.18.

The overall performance on this question was good. About 55 per cent of the candidates were able to score full marks.

In Part (a), the majority of candidates attempted to complete the square but only about 60 per cent were able to do so successfully. The majority of the candidates used the classical method to complete the squares and about 95 per cent of them were able to identify that $a = 3$, however, finding correct values for *b* and *c* proved to be difficult due to numerical errors they made along the way. Some candidates used other techniques to arrive at the values of their constants; $3x^2 - 9x + 4 \equiv a(x + h)^2 + k$, where $a = 3$, $h = \frac{b}{2a} = -\frac{3}{2}$ and $k = c - ah^2$ or $\frac{4ac - b^2}{4a} = -\frac{11}{4}$ $\rightarrow$ $3x^2 - 9x + 4 \equiv 3\left(x - \frac{3}{2}\right)^2 - \frac{11}{4}$ being the more common one. Some of the candidates using this approach did not remember the formulae correctly; $h = -\frac{b}{2a} = \frac{3}{2}$ and $k = \frac{b^2 - 4ac}{4a} = \frac{11}{4}$ were some of the common mistakes made.
The majority of candidates was not able to provide the coordinates of the minimum point from their completed square form. They gave instead either the minimum value of the function or the *x* value at the minimum point only.

In Part (b), deriving the sum and the product of the roots from a given equation was well known but some candidates confused the signs in determining their answers; $\alpha + \beta = \frac{b}{a}$ and $\alpha\beta = \frac{-c}{a}$ were sometimes used. Other candidates used the numerical values they obtained by solving for α and β instead of α + β and αβ. It was also difficult for some candidates to express $\frac{1}{\alpha}+\frac{1}{\beta}$ in terms of $\alpha + \beta$ and $\alpha\beta$.
In Part (c), many candidates experienced great difficulty in simplifying the equation they obtained after eliminating one of the variables from the given equations. Some also did not proceed to find the coordinates of the points leaving only the *x* values they obtained. In many cases where they proceeded to find *y*, errors were often made.

In Part (d), candidates displayed a lack of knowledge about arithmetic progressions and geometric progressions when given in a real world context. Many attempted a trial and error approach rather

than using an arithmetic progression where $T_n = a + (n-1)d$ and some even used a geometric progression. Some candidates used the first term as 38 400 instead of
36 000 while others used the formula for the sum of terms, $S_n = \frac{n}{2}(2a + (n-1)d)$.

Candidates need to be more vigilant when using equations and numerical figures given in the question paper in their solutions. Many of such errors were seen throughout the question.

**Solutions**

(a) (i) $3\left(x - \frac{3}{2}\right)^2 - \frac{11}{4}$ (ii) Minimum point is $\left(\frac{3}{2}, -\frac{11}{4}\right)$

(b) $\frac{1}{\alpha} + \frac{1}{\beta} = -\frac{3}{2}$

(c) Points of intersection are (−3, 37) and $\left(-\frac{5}{2}, \frac{63}{2}\right)$

(d) Annual salary for the 9th year = \$55 200

**Section 2: Coordinate Geometry, Vectors and Trigonometry**

<u>Question 3</u>

This question tested candidates' ability to:

- Determine the coordinates of the centre of a circle and (ii) the length of its radius given the equation in the form $x^2 + y^2 + 2fx + 2gy + c = 0$; (iii) determine the equation of the normal to a circle at a given point on the circle.
- Determine the unit vector in the direction of $\overrightarrow{AB}$ given two vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ written in the form ***a*i** + *b**j***; (ii) calculate the acute angle between the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

There were 2923 responses to this question. The mean mark was 6.03 with a standard deviation of 4.03.

Candidates' recall of knowledge of coordinate geometry and vectors was good.

In Part (a) (i), candidates were able to determine the coordinates of the centre of the circle by either completing the square or by using coordinates of centre $=\left(\frac{-2f}{2}, \frac{-2g}{2}\right)$, from the equation of the circle. The approach involving the coefficient method was the most popular one used by candidates.

In Part (a) (ii), finding the length of the radius was a challenge to most candidates. This was often the only mark lost in this section.

In Part (a) (iii), candidates were able to apply the gradient algorithm using the coordinates of the centre they obtained. However, some of them misinterpreted this gradient to be that of the gradient of the tangent and proceeded to obtain the gradient of the normal as the negative reciprocal of this result.

There was a unique solution of determining the **equation of the normal:**
$(y_1 + g)x - (x_1 + f)y - gx_1 + fy_1 = 0$

In Part (b) (i), candidates correctly obtained the vector $\overrightarrow{AB}$. Most candidates correctly calculated the modulus of $\overrightarrow{AB}$ but were unable to write the **unit vector** $\overrightarrow{AB}$

In Part (b) (ii), candidates were able to determine the dot product and the moduli of $\overrightarrow{OA}$ and $\overrightarrow{OB}$. However, errors in computations led to an incorrect value of the required angle.

**Solutions**

(a) (i) coordinates of centre are (6, 11)

(ii) radius = $\sqrt{5}$

(b) (i) unit vector $\overrightarrow{AB}$ = $\frac{1}{\sqrt{29}}(2i-5j)$

(ii) $A\hat{O}B = 20.8^0$

Question 4

This question tested candidates' ability to:

- Determine the area in a circle between a chord and an arc given the formula for the area of a sector.
- Use the trigonometric identity $\sin^2\theta = 1-\cos^2\theta$ to solve a quadratic equation with trigonometric arguments.
- Prove a trigonometric identity using the expansions for sin2θ and cos2θ.

Candidates performed best on Part (a), which required finding the area of the shaded region. Most candidates knew the formula for finding the required area. However, many candidates substituted $30^\circ$ for the angle in the formula that involved radians.

In Part (b), many candidates knew that they had to substitute $\sin^2\theta = 1-\cos^2\theta$. Most of the candidates who substituted correctly were able to simplify and form the quadratic equation. Many of them were able to solve the quadratic equation and obtain the required angles.

In Part (c), most candidates received marks for substituting correctly for $\sin 2\theta = 2\sin\theta\cos\theta$. However, many did not realize that the substitution of $\cos 2\theta = 2\cos^2\theta - 1$ was necessary to lead to the proof. Some of those who substituted correctly had difficulty in factorizing and simplifying and as a result were unable to complete the proof.

**Solutions**

(a) Area of shaded region = $\frac{4}{3}\pi - 4 \ cm^2$

(b) $\theta = 104.5^\circ, 255.5^\circ$

**Section 3: Introductory Calculus**

Question 5

This question tested candidates' ability to:

- Use the product rule to differentiate the product of a quadratic expression and a trigonometric function of the form sin *ax* where $a \in Z$ .
- Determine the stationary points of a polynomial of order 3; determine the nature of the stationary points.
- Use the concept of connected rates to determine the rate of change of the radius of a spherical balloon w.r.t. time, given the rate of change of the volume w.r.t. time and the formula for the volume of a sphere.

There were approximately 2891 responses to this question. The mean mark was 6.35 with standard deviation of 4.06.

Candidates' understanding of the algorithmic procedures in the Introductory Calculus and Coordinate Geometry content in this section of the syllabus was generally good. However, most candidates were challenged by the real-life application of the rate of change in Part (c).

In Part (a), candidates easily obtained the derivative of $2x^2 + 3$ to be $4x$. Their responses were often incorrect for $\frac{d \sin 5x}{dx}$. The coefficient 5 was often omitted and in a few instances $-5$ or $\frac{1}{5}$ were presented as coefficients of cos 5*x*. In addition, many candidates left out the coefficient, 5, which was part of the argument of the sine function. Candidates correctly applied the product rule but the mark for simplifying their result was rarely obtained.

In Part (b), differentiating the function $f(x) = x^3 - 5x^2 + 3x + 1$ to obtain the first derivative $f'(x) = 3x^2 - 10x + 3$ and equating the resulting quadratic equation to zero to obtain the coordinates of the stationary points are successfully done. Candidates also correctly differentiated $f'(x)$ to obtain the second derivative; $f''(x) = 6x - 10$ which enabled them to correctly determine the nature of each of the stationary points. However, a few of them came to the wrong conclusions based on the signs of the second derivative; at the stationary points. Some of them concluded that a negative value of the second derivative at the stationary point resulted in a minimum point and a positive value resulted in a maximum point.

In Part (c), candidates did not show full understanding of the terminology for the derivatives. Assigning 200 $cm^3\ s^{-1}$ correctly to $\frac{dV}{dt}$ was not commonly seen and correct substitution into derivatives was not often done. In addition, formulating the chain rule was also challenging and hence the algorithmic mark for the correct result was often lost.

**Solutions**

(a) $(10x^2 + 15)\cos 5x + 4x \sin 5x$

(b) (i) Coordinates stationary points are $\left(\frac{1}{3}, \frac{40}{27}\right)$ and $(3, -8)$

(ii) Maximum point is $(3, -8)$; Minimum point is $\left(\frac{1}{3}, \frac{40}{27}\right)$

(c) $\frac{dr}{dt} = \frac{1}{2\pi}$

Question 6

This question tested candidates' ability to:

- Evaluate a definite integral for a trigonometric function with a multiple angle.
- Equate the derivative of a function to a given value to determine the value of an unknown constant.
- Formulate the equation of a curve given its gradient function and a point on the curve.
- Find the volume obtained by rotating a finite region, in the first quadrant bounded by a curve and the *x*-axis and the lines $x = 0$ and $x = 1$, through 360°.

Part (a) was poorly done, very few of the candidates were able to integrate $\cos 3\theta$ correctly. Those who remembered that the integral of $\cos\theta$ is $\sin\theta$ often gave the integral of $\cos 3\theta$ as $\sin 3\theta$.

In Part (b) (i), approximately 65 per cent of the candidates were able to correctly calculate the value of the unknown constant, *k*, by substituting $x = 2$ and equating the result to 14. A few of them substituted the value $y = 3$ for $\frac{dy}{dx}$. This clearly demonstrates very superficial understanding of the topic. However, the majority of the candidates was able to use their value of *k* to proceed to Part (b) (ii).

In Part (b) (ii), about 50 per cent of the candidates incorrectly used the equation of a line $y = mx + c$, as the equation of a curve. Candidates who correctly calculated the value of *k* sometimes had difficulty integrating correctly. In a few cases, the constant of integration was left out, which resulted in incorrect equations.

Part (c) was generally well done, with most of the candidates knowing the volume of revolution formula, as well as how to obtain the limits of integration. However, incorrect expansions of $y^2$, given $y = x^2 + 1$ was frequently observed, $x^4 + 1$ or $x^4 + 2x^2 + 2$ were some of the expansions seen. Some candidates were very careless when integrating, they either forgot to integrate the constant term or integrated the variable terms incorrectly. Simple addition of fractions such as $\frac{1}{5} + \frac{2}{3} + 1$ proved challenging for some candidates. Candidates also gave their answers in decimal form which reflected a lack of knowledge of exactness.

**Solutions**

(a) $-\frac{1}{3}$

(b) (i) $k = 7$

(ii) Equation of the curve is $y = \frac{7}{3}x^3 - \frac{7}{2}x^2 - \frac{5}{3}$

(c) Volume generated = $\frac{28}{15}\pi$ units$^3$

**Section 4: Basic Mathematical Applications**

Question 7

This question tested candidates' ability to:

- Use the concept of a possibility space to solve problems involving probability.
- Calculate the mean for grouped data.
- Construct a tree diagram and use it to calculate probabilities.

This question was the least popular of the optional questions with about 1530 candidates attempting it. It was poorly done with approximately 70 per cent of the candidates scoring under 10 marks, however, there were 29 candidates scoring full marks.

Candidates performed poorly on Part (a), which required finding the probability involving three independent events. Most candidates were able to identify that they were independent events. A few of the candidates worked with two events instead of the three events that were given because they were unable to correctly define the possibility space for the question.

Candidates performed best on Part (b), many of them knew that they had to calculate the midpoints in order to calculate the mean. However, far too many candidates could not do this part which is also covered in the CSEC General Mathematics syllabus.

In Part (c), the majority of the candidates received most of their marks from Part (c) (i), which required drawing a tree diagram. Some of the candidates did not label the branches properly; for example, they did not show their probabilities on the branches, while others did not label the days. Some of the candidates did not have the percentage sign (%) or the decimal equivalent for their probabilities. Most candidates knew that total probabilities on their branches should add to 1 and the majority of them attempted to use their tree diagram to calculate the probability of rain falling on Wednesday.

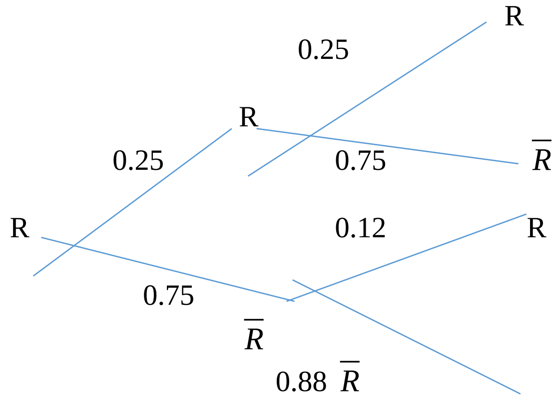
**Solutions**

(a) (i) Probability motorist has to stop at only one traffic light = 0.42

(ii) Probability motorist has to stop at at least two traffic lights is = 0.5

(b) Mean = $\frac{340}{25} = 13.6$

(c) (i) Monday Tuesday Wednesday




Key: R means it rains; $\bar{R}$ means it does not rain

(ii) Probability it rain on Wednesday = 0.1525

Question 8

This question tested candidates' ability to:

- Draw and make use of a velocity–time graph for rectilinear motion to find acceleration and increase in displacement.
- Apply the result $v = \int a\mathrm{d}t$ and use it to calculate the velocity function, given the velocity at a specified time and use the result $s = \int v\mathrm{d}t$ to determine the displacement of a particle at a given instance of time.

There were approximately 2298 responses to this question. The mean mark was 10.27 with a standard deviation of 5.59. Two hundred sixty-three candidates obtained full marks
(20 marks).

Candidates performed best on Parts (a) (i) and (ii). They were able to draw the correct velocity–time graph for the given information, as well as calculate the acceleration of the particle.

They used a variety of methods to determine the displacement in Part (a) (iii); some calculated the displacement by finding the area of a trapezium, others found the area of a rectangle plus the area of a triangle while others used equations of motion formulae or determined the equation of the velocity—time graph and integrated between the limits $t$ = 0 and $t$ = 4. Approximately 25 per cent of the candidates only calculated part of the area under the graph.

The majority of the candidates who attempted Part (a) (ii), knew that the acceleration could be obtained using the formula . $\text{acceleration} = \dfrac{\text{change in velocity}}{\text{change in time}}$. However, instead of using the initial and final velocities and the corresponding times, a few of the candidates used non- exact values which led to incorrect answers.

Parts (b) (i) and (ii) presented the most problems for more than half of the candidates. Many of them used equations of motion, which are only valid when the acceleration is constant, to do these parts. In Part (b), $a = 3t - 1$ was given and candidates were required to obtain an expression for the velocity at the instant when the time was 4, and the displacement from the starting point at ***t*** = 3. Those who knew they had to integrate the velocity function to obtain the displacement sometimes forgot to include their constant of integration, which had to be calculated using the given information. This led to incorrect answers for both the velocity and displacement.

**Solutions**

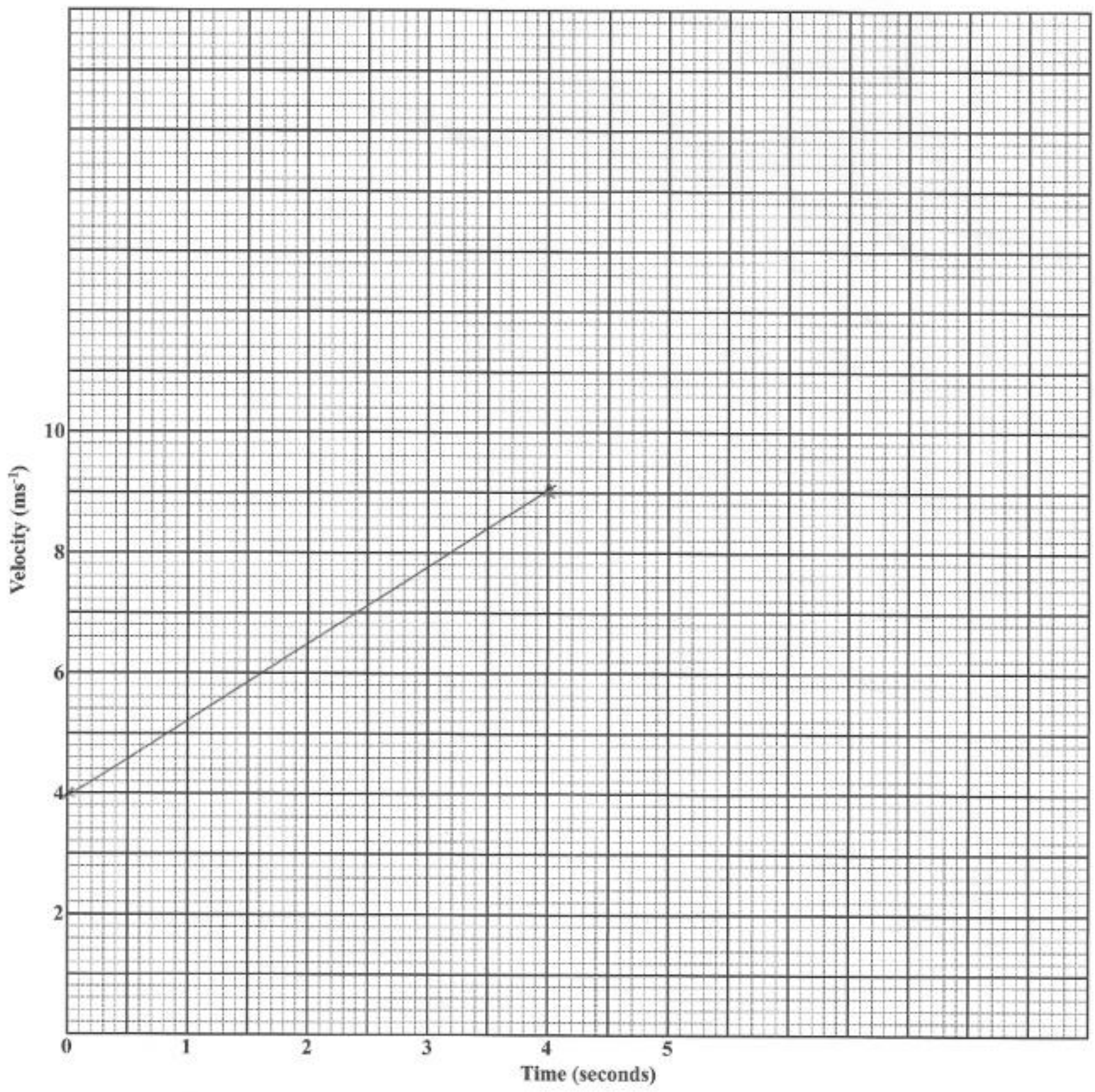
**Recommendation for Teachers**:

- Students must be aware of the mathematical jargon associated with kinematics, for example, motion in a straight line (rectilinear motion) invariably means that the acceleration is constant.
- Students should be encouraged to use the information given in plotting a straight line graph to obtain the gradient, instead of choosing their own non-exact values.
- Where graph paper with the axes already written is provided, teachers should insist that that graph paper be used instead of any other graph paper.
- Students should be informed that the equations of motion (i) $v = u + at$ (ii) $s = ut + \frac{1}{2}at^2$ and (iii) $v^2 = u^2 + 2as$ should only be used if the acceleration is constant.

- Teachers need to give a greater number of examples and practice questions after completing topics so that their students can get a clearer grasp of the concepts.

**Solutions**

(a) (i) See graph below
(ii) Acceleration = 1.5 m $s^{-2}$
(iii) Displacement = 24 m
(b) (i) Velocity = 20 m $s^{-1}$ (ii) displacement = 13 m



## Paper 031 — School Based Assessment (SBA)

The marking team moderated many more projects this year from 191 examination centres, a further indication of the increase in the number of new schools as well as studnets pursuing studies in Additional Mathematics. Many of the projects submitted were of a high quality and generally related to real-world situations. The mean mark awarded on the SBAs was 32.08 (out of a possible total of 40 marks), with a standard deviation of 6.33, compared with 33.06 and 6.30 respectively in 2014. Many of the sample submissions received full marks.

**Comments specifically for Project A:**

**<u>Title</u> — *what is the project investigating?***

- Many of the students gave a name for the project rather than a title, these needs to be more specific.
- In most cases, students related to real world problems.

**<u>Purpose of Project</u> — *why this project/investigation is being done?***

- In many cases, the purpose was a replica of the title.
- Few students were specific enough about the purpose of the project.

**<u>Mathematical Formulation</u> — *how the problem described in the purpose is going to be investigated/solved***

- Generally, most students correctly identified all the important elements of the problem and showed understanding of the relationship between elements.
- In many cases, students did not use appropriate additional mathematical methods/models to solve the problems.
- The content used in some cases reflected Form 3/Grade 8 level mathematics, for example, some projects had only simple calculations with area and volume.

**<u>The Problem Solution</u> — *here the problem is solved***

- The assumptions were not clearly stated (must be explicit and not simply implied). However, some students stated limitations of the project.
- Explanations were not sufficient and not placed between every successive step in the problem solution.
- Calculations were precise but solutions were not clearly stated.

<u>**Application of Solution**</u> **—** ***here the solution derived is applied to the real–world context***

- This section was fairly well done. However, many students were unaware of how to show that the solution or proof of the given problem was valid.
- Validation can be achieved by an alternative method or by comparison to a similar study.

<u>**Discussion of Findings/Conclusion**</u> **—** ***interprets the solution and relates it to the purpose of the study***

- The discussions were too general and in most cases the findings/solutions were not related to the purpose of the project.
- The conclusions in many instances were found not to be valid. Some students in made many conclusions but not all were related to the purpose of the project.
- Suggestions for future analysis can also include the use of this study in related areas or improvements for further studies.

<u>**Overall Presentation**</u>

- There was a marked improvement in the correct use of spelling and grammar.
- Students used the correct mathematical jargon and symbols appropriately most of the time.

**Areas of Strength**

- Many of the submissions were of a high quality and related to real-world situations to which mathematical concepts were applied. A few were unique and interesting and represented the investigation of good ideas.
- There was evidence indicating that students applied a high degree of effort to the task. They understood the focus of their topic selection and conducted the requisite research thoroughly.

**Areas of Weakness**

- The CSEC Additional Mathematics rubric was absent from the samples submitted by some centres.
- Some SBA project titles and aims were not clear, not concise and not well defined.
- Teachers must be alert to any suspected plagiarism. Plagiarism must not be condoned and projects should be marked accordingly.
- The majority of students did not incorporate or suggest any future analysis in their submissions.
- Some students failed to adequately connect their finding(s)/conclusion(s) to their aim(s), their data collected and their analysis of the data.

**Comments Specifically for Project B**

**<u>Title</u> — *what the project is investigating?***

- Many of the students gave a name for the project rather than a title.
- In most cases, the students related to real–world problems.

**<u>Purpose of Project</u> — *why this project/investigation is being done?***

- In many cases, the purpose was a replica of the title.
- Few students were specific enough about the purpose of the project
- The majority of the students was **not** able to identify the variables in the project properly.

**<u>Method of data collection</u> — *how the data is being collected***

- Most students were able to describe the data collection method clearly.
- It is expected that students mention the type of data collected (primary, secondary) and/or sampling used for example, random sampling, stratified sampling. This occurred only in a few instances.

**Presentation of data**

- Most studdnts had a minimum of one table or chart.
- Most students used the graphs, figures and tables appropriately in their presentation.

**Mathematical Knowledge (Analysis of Data)**

- Mathematical concepts in many cases were not used appropriately from the Additional Mathematics syllabus
- In many cases, students used concepts and representations which reflected Form 2 level Mathematics. For example, some students used only pie charts and bar charts.
- In many instances, the analysis was attempted, but students were not able to use it in their discussions. For example, standard deviation was calculated but was not used in the discussion (there was no mention of spread or skewness and what they mean? in relation to the project).
- Students were not too clear on the two approaches to be used in the analysis.
- In many cases the two approaches used were of the same concept. For example, mean, mode and median are all measures of central tendencies.

**Discussion of Findings/Conclusions**

- In some instances there was no statement of findings. However, most statements made were linked to the solution of the problems.
- Suggestions for future analysis can also include the use of this study in related areas or improvements for further studies.

**Overall Presentation**

- There was a marked improvement in the correct use of spelling and grammar.
- Students used the correct mathematical jargon and symbols most of the time.

## Paper 032 — Alternative to SBA

This paper tested candidates' ability to:

- Determine the maximum profit for a given profit function which was quadratic.
- Represent real–world situations using quadratic equations and solve for unknown variables.
- Derive an expression, in algebraic form, for the area of a composite figure given its dimensions.
- Determine the range of values of an unknown side for which the area of the shape exists.

There were 83 candidates who wrote this paper, 81 of them attempted the question. The mean mark was 5.26 with a standard deviation of 4.46 compared with 8.69 and 7.75 respectively in 2014. No candidate received full marks (20 marks). The highest mark recorded was 18 out of 20.

This paper was very poorly done. Only about 3.7 per cent of candidates were able to achieve 75 per cent and above. Of the three parts, which comprised this paper, Part (b) was done the best. Candidates need to know the functions of calculus and when it should be used as opposed to using normal algebraic procedures.

In Part (a), candidates were required to use differentiation, or some other suitable method, to determine the maximum daily profit of a company. Many candidates tried to solve for $x$ by factorizing the expression, completing the square or using the quadratic formula. Obviously, candidates did not connect differentiation with this question. After this part, the majority of candidates understood that substitution was necessary to determine the number of hundred dollars needed, even if the value they substituted was obtained erroneously. In Part (a), some of the candidates who solved the given equation obtained the values $x = -30$ and $x = 10$. They then concluded that $x = 10$ would give the maximum profit of $1000.

**Common Errors**

Candidates solved for the roots of the equation rather than differentiate and equate to 0.

Candidates did not multiply by 100 to get the required dollar value for their answer.

In Part (b), candidates were required to determine dimensions of a square and rectangle given specific conditions. This part was relatively well done compared with the other two parts. Expressions for the dimensions of the square and rectangle were often correctly obtained, thus $A = x^2 + (x+3)(x+6)$ was usually given as the total area although there were many variations presented such as $w = s + 3$ and $l = w + 3$, where $s$ represented the length of the square, $w$ represented the width of the rectangle and $l$ the length of the rectangle. The candidates who represented the total area correctly invariably obtained a quadratic equation and even though some factorized incorrectly, they were able to apply their answers to produce acceptable dimensions for each shape. Some candidates used the trial and error method to arrive at the correct dimensions. This method was not penalized once supporting statements were seen.

In Part (c) (i), candidates were required to obtain an expression for area in terms of $x$ only. This part was poorly done and it was seen that candidates could not apply the formula for the area of a triangle to algebraic processes. Many knew they needed to add the areas of the rectangle and triangle, but were confused by what should be done next. Many candidates did not recognize the need to use Pythagoras' theorem in representing $h$, the height of the isosceles triangle in terms of its base, $x$, and its slant side which was 4 cm. In cases where Pythagoras' theorem was used, it was incorrectly applied.

In Part (c) (ii), candidates were required to find the required domain for area. Very few candidates even attempted this part and it was clear that they could not link area in Part (c) (i) to the length of a side, $x$, even though this was stated in the question. Additionally, candidates could not relate the need for $x$ to be positive in order to represent length. Candidates also did not use the fact that the height, $h$, of the triangle had to be positive in order to find the required range for $x$ using the result $h = \sqrt{\frac{64 - x^2}{4}} > o$ which would imply that $\sqrt{64 - x^2} > o$ .

With respect to candidates' responses on this question, it is recommended that candidates be given more practice questions. This will allow them to apply theory to real situations.

**Solutions**

(a) Maximum daily profit = \$10 000

(b) (i) Dimensions of the square are 5.5 cm by 5.5 cm
(ii) Dimensions of the rectangle are 11.5 cm by 8.5 cm

(c) (i) Area of shaded surface $= 4x + \frac{x\sqrt{64 - x^2}}{4}$

(ii) Domain of $x$ $\{x : 0 < x < 8, x \in \mathbf{R}\}$

Generally, teachers need to ensure that the syllabus is completed with sufficient time to give students time to do practice exercise and past papers.

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

# REPORT ON CANDIDATES' WORK IN THE CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION

MAY/JUNE 2016

# ADDITIONAL MATHEMATICS GENERAL PROFICIENCY EXAMINATION

## General Comments

This year, 2016, saw the 5$^{th}$ sitting of candidates for Additional Mathematics. This year also saw the transition of this subject to being marked on an e-marking platform, joining the other CXC/CSEC subjects being so marked.

As established, the examination consists of three papers:

- Paper 01 – a 45-item multiple choice paper
- Paper 02 – a structured, 'essay-type' paper consisting of eight questions
- Paper 031 or Paper 032 – Paper 031 is a School-Based Assessment (SBA) project for candidates in approved educational institutions, whilst Paper 032 is an alternative to the SBA for out-of-school (private) candidates

Questions in the examination targeted the content and specific objectives of the Additional Mathematics syllabus (CXC 37/G/SYLL 10) with the aim of ensuring a wide coverage of the syllabus. The questions were designed at the appropriate level to test the following skills: Conceptual Knowledge, Algorithmic Knowledge and Reasoning.

Paper 01 tests content from Sections 1, 2 and 3 of the syllabus. Paper 02 tests content from all four sections of the syllabus. As before, this year the paper was structured around these four sections with each section containing two problem-solving questions. The questions in Sections 1, 2 and 3 were all compulsory. Each question in Sections 1 and 3 was worth 14 marks, while each question in Section 2 was worth 12 marks. Section 4 consisted of two questions, one on Data Representation and Probability and the other on Kinematics and each question was worth 20 marks. Candidates were required to answer only one question from this final section. Paper 031 is the SBA component of the examination. Candidates are required to do one project chosen from two types of projects: a mathematical modelling project (Project A) and a data handling/statistical analysis project (Project B). The SBA component is worth 20 marks. Private candidates can sit an alternative paper to the SBA, Paper 032, which consists of one in-depth, extended question from Sections 1, 2 and/or 3 of the syllabus. This paper is worth 20 marks. Again this year, a formula sheet was included as part of the question paper on Papers 01, 02 and 032.

This year the trend of increasing number of candidates registering for this subject continued with 4573 candidates registered, a 14 per cent increase from the 4016 candidates in 2015. Over the five years of the subject being examined, candidate numbers have increased by approximately 160 per cent.

With respect to overall performance, 68 per cent of candidates earned Grades I–III, compared with 73 per cent in 2015.

## Detailed Comments

### Paper 01 – Multiple Choice

This was a 45-item paper covering Sections 1, 2 and 3 of the syllabus. The mean and standard deviation on this paper were 35.55 and 13.03 respectively (weighted up to a total of 60 marks), compared with 39.54 and 13.36 in 2015. No candidate achieved full marks.

### Paper 02 – Structured Essay Questions

This paper consisted of eight questions, of which Questions 1 to 6 (Sections I – III) were compulsory. Candidates had to choose either Question 7 or Question 8 in Section IV. The maximum possible score on this paper was 100 marks. The mean and standard deviation for this paper were 44.05 and 25.02 respectively, compared with 47.43 and 26.99 in 2015.

**Section 1 – Algebra and Functions**

<u>Question 1</u>

This question tested candidates' ability to:

- State the range of a given linear function, sketch graphs of said function and its inverse and state the relationship between the two;
- Solve logarithmic equations as well as use logarithms to solve algebraic equations/change subject of formula.

The mean mark on this question was 6.22 with standard deviation of 3.78. The mean represented 44.4 per cent of the maximum possible 14 marks.

This question was generally not well done by candidates. The areas of good performance were Part (a) (i), determining the range of the function, and (a) (ii), finding $f^{-1}(x)$. Even so, although most candidates knew how to substitute domain values in (a) (i) to find co-domain values, they did not demonstrate an understanding of how to state the range of the function for the given domain values using inequality signs, and $-9 \leq x \leq 3$ was often given. For Part (a) (iii) many candidates had difficulty sketching the two graphs correctly, particularly the graph of $f^{-1}(x)$. Further, and possibly as a result of errors in this part, for Part (a) (iv) the relationship between the two graphs was not well known.

For Part (b), many candidates recognized that the use of a substitution would make the equation easier to solve but poor manipulation using the substitution resulted in many erroneous solutions.

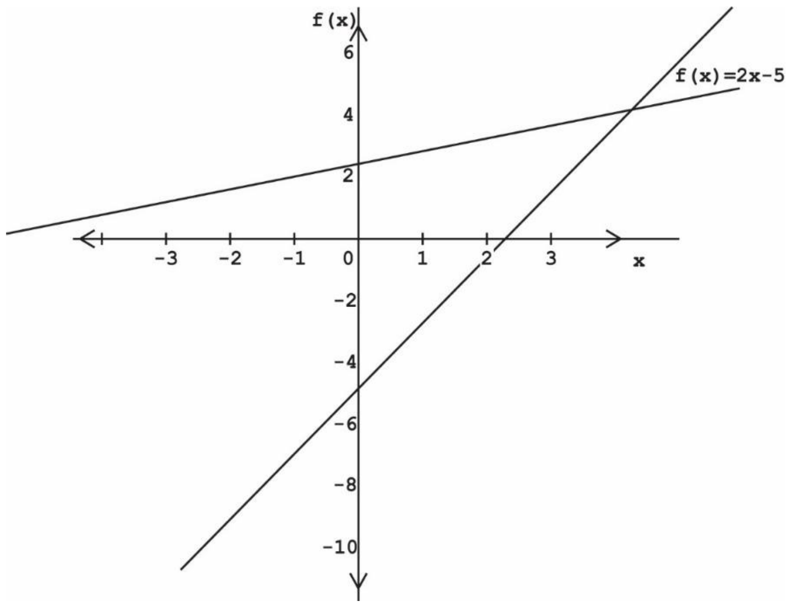
For Part (c) (i), the majority of respondents failed to recognize that the use of logarithms would be required to make *c* the subject. For Part (c) (ii), there was a good understanding of the laws of logarithms. However, the incorrect application of these laws was frequently evident.

Teachers are asked to give some attention to the writing of the range of functions in the inequality format. Also, teachers are asked to provide candidates with more practice on changing the subject of formulas where the use of logarithms is necessary, that is, in recognition of these question types and practice in their use, as this question tested here was poorly done.

**Solutions**

(a) (i) Range = {−9, −7, −5, −3} (ii) $f^{-1}(x) = \frac{x + 5}{2}$

(iii)




(iv) The graphs are reflections of each other in the line *y* = *x*.

(b) $x = -1$

(c) (i) $c = \frac{h \log p}{\log T - \log k}$ (ii) $x = \frac{-5}{4}$

<u>Question 2</u>

This question tested candidates' ability to:

- Determine the nature of the roots of a quadratic equation and sketch the graph of said function;
- Calculate the sum of a geometric series and convert a real-world problem to a solvable mathematical problem using arithmetic progression (that is, obtain expressions for the general terms and sum of an arithmetic progression in order to solve a real-world problem)

The mean mark on this question was 5.10 with standard deviation of 3.16. The mean represented 36.4 per cent of the maximum possible 14 marks.

Generally, this question was not done well by candidates. Many candidates did not appear to understand what was being asked in Part (a) (i), in that many solved the quadratic equation stating the roots but did not go on to describe the *nature* of these roots. In Part (a) (ii) candidates for the most part were able to draw a reasonable sketch of the given function. However, they lost marks for not identifying the *f(0)* point as −9 or some other critical points being left off.

For Part (b), most candidates did not recognize that the equation given was a geometric progression as it was left blank or solved as if it were an arithmetic progression. Some candidates, however, were able to solve it by correctly placing the values directly into their calculators and summing to the number of terms given. Whilst this was not the most efficient way of solving the question given, it did show a level of understanding of what was being asked and as well knowledge of an inelegant route to a solution.

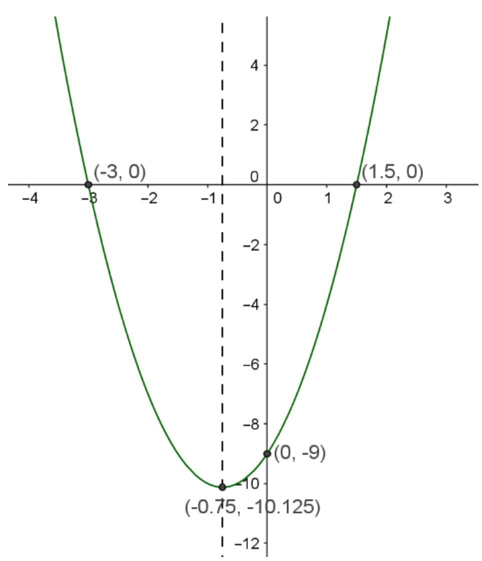
In Part (c) the majority of candidates recognized that there was a difference of 15 between successive terms and knew to use the sum of terms of an arithmetic progression. However, only a few candidates deduced the correct first term of $25 (rather than $85 which was more widely used), and also the correct number of terms. Many candidates had difficulty understanding when the arithmetic progression started and that the terms given in the problem statement were not the first three terms.

Teachers are asked in the teaching of solving of quadratic equations to further examine the *nature* of these roots here and as well what they mean with respect to the look of the quadratic graph. It is also recommended that teachers discuss key points that should be included in the sketch of a graph such as the minimum point and axis intercepts. As well, candidates need more exposure to and practice in translating real-world word problems into solvable mathematical forms, especially as relates to the use of arithmetic and geometric progressions.

**Solutions**

(a) (i) Equation has two real and distinct roots (since $b^2 - 4ac > 0$)

(ii)




(b) $\sum_{1}^{25} 3^{-n} = \frac{1}{2}$

(c) $6370.00

**Section II: Coordinate Geometry, Vectors and Trigonometry**

Question 3

This question tested candidates' ability to:

- Find the equation of a circle given the coordinates of the end points of a diameter and to find the equation of a tangent to the circle at a particular point;
- Determine the position vector *OP* of a point which divided a vector *AB* in a stated ratio given the two position vectors *OA* and *OB*.

The mean mark on this question was 5.73 with standard deviation of 4.17. The mean represented 47.8 per cent of the maximum possible 12 marks.

In Part (a) (i), most candidates were able to correctly find the midpoint of the circle, some by inspection but most from the formula $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$, however some experienced difficulty finding the length of its radius, *r*, or writing the equation of the circle correctly. In particular, those who used the form $x^2 + y^2 + 2fx + 2gy + c = 0$ were generally unable to correctly obtain the value of *c*. In Part (ii) many candidates knew that the radius (diameter) is perpendicular to the tangent at the point of tangency and correctly used this result to obtain the gradient of the tangent as the negative reciprocal of the gradient of the normal (diameter or radius) and hence obtained the equation of the tangent at the stated point or another reasonable point.

Part (b) was generally fairly well done, and a number of novel solutions were seen here, demonstrating that candidates had a good understanding of what was being asked. Common errors saw candidates finding PO and giving this as the same as OP and also some candidates not getting the correct final answer due to numerical errors made along the way.

Teachers should ensure that candidates understand how to compute the equation of the circle in the various forms and obtain the necessary parameters. It also seems that some candidates have misunderstandings regarding how to compute certain vectors.

**Solutions**

(a) (i) Circle equation is $(x-1)^2 + (y-3)^2 = 5$

(ii) Various, depending on points used; one possible based on question paper is $y = \frac{2}{3}x + \frac{20}{3}$

(b) $\overrightarrow{OP} = \begin{pmatrix} -2 \\ \frac{17}{4} \end{pmatrix}$

Question 4

This question tested candidates' ability to:

- Find the area of a shaded region formed by two sectors of different radii using the appropriate formula;
- Use the compound angle formula to determine, in surd form cosine 105°;
- Prove a trigonometric identity.

The mean mark on this question was 4.05 with standard deviation of 3.75. The mean represented 33.8 per cent of the maximum possible 12 marks.

Candidates performed best on Part (a), which required them to find the area of a shaded region formed by two sectors of circles of different radii where one sector was nested in the other. Generally, candidates realised they needed to subtract the smaller sector area from the larger, and too were able to set up the expression for this. However, many candidates were unable to simplify the resulting algebraic expression, that is, $\frac{1}{2}(x+2)^2 \times \frac{2\pi}{9} - \frac{1}{2}x^2 \times \frac{2\pi}{9}$ to obtain $\frac{\pi}{9}(4x + 4)$ or $\frac{4\pi}{9}(x+1)$, while some candidates converted the angle from radians to degrees while still using the formula $\frac{1}{2}r^2\vartheta$ for the area of the sector. Responses such as $80x + 80$ were sometimes seen where $\frac{4\pi}{9}$ radians = $80^0$ was used.

In Part (b), many candidates knew that they had to manipulate the angles, 30°, 60° and 45° to find an expression for 105°. However 105° = 60° + 45° was not seen often enough. A common error seen was expressing cos $105^0$ as cos $30^0$ + cos $30^0$ + cos $45^0$, that is, treating the cosine function as a linear function. Also seen was cos $105^0$ expressed as 2 cos $30^0$ + cos $45^0$. While it is possible to obtain the correct answer from this last approach, candidates generally incorrectly applied linear algebra to cos $2 \times 30^0$ and so did not correctly complete the question. Additionally, candidates were not always able to correctly simplify the surds they used, as asked for in the question, and attempted to use a calculator, though no marks could be awarded for simply writing the answer as a decimal. There were a few novel solutions where candidates recognized that 105° = 180° – 75°, then used the compound angle formula for cosine to obtain – cos 75°, and used a further application of the compound angle formula with 45° and 30° to arrive at the correct solution.

In Part (c), the majority of the candidates started from the left-hand side and correctly used the compound angle formula for the sine of a sum of angles but often omitted the step showing the split which would have led to the final result. A few candidates started from the right-hand side but most of them did not realise that the reverse of the compound angle formula for sines would have led them to the correct conclusion.

Teachers are asked to encourage candidates, when proving identities, to attempt to use the harder side of the equation to prove the easier side and to remind them that all parts of the mathematical functions are used like in this case the concept of fractions was needed. Candidates should also have more practice with manipulation of surds and understanding how to rewrite an angle into a combination of known angles.

**Solutions**

(a) $\frac{4\pi}{9}$ $(x + 1)$ units$^2$

(b) $\frac{\sqrt{2}-\sqrt{6}}{4}$

(c) $\frac{\sin(\theta+\alpha)}{\cos\theta\cos\alpha} = \frac{\sin\theta\cos\alpha+\cos\theta\sin\alpha}{\cos\theta\cos\alpha}$

$= \frac{\sin\theta\cos\alpha}{\cos\theta\cos\alpha} + \frac{\cos\theta\sin\alpha}{\cos\theta\cos\alpha}$

$= \tan\theta + \tan\alpha$

**Section III: Introductory Calculus**

<u>Question 5</u>

This question tested candidates' ability to:

- Use the chain rule to differentiate a given algebraic expression and simplify their answer;
- Determine the equation of a normal to a curve at a point on the curve;
- Determine equations for tangents to a curve from points on the curve.

The mean mark on this question was 5.28 with standard deviation of 4.40. The mean represented 37.7 per cent of the maximum possible 14 marks.

In Part (a), most candidates knew to rewrite the equation with the power of $\frac{1}{2}$ and were able to differentiate that part of the equation. Further, many candidates also knew they needed to use the chain rule, although this was not always correctly applied. There were also a number of candidates who could not simplify their correctly obtained expression of $5x(5x^2-4)^{-\frac{1}{2}}$ back into a square root form, which was a requirement of the question.

For Part (b), many candidates expanded the function and were able to correctly find $\frac{dy}{dx}$ and the gradient of the tangent at $x = 1$. However, far too many appeared not to recognize that this value was the gradient of the tangent and so used this gradient to find an equation (that is, the equation of the tangent) rather than using the obtained value to find the gradient of the normal and hence the equation of the normal as required. There were a few candidates who opted to differentiate using the product rule rather than simplifying the expression given first. Whilst this is possible, it was a more difficult method and at times led to errors in the answer for the derivative. Candidates should be encouraged to look at different methods and determine which one is the simplest before proceeding.

For Part (c), many candidates were able to correctly differentiate $y = x^2$. However, even though the question asked for equations of TWO tangents at the points where $y = 16$, a fair proportion of candidates still only went on to find one equation of tangent where $x = +4$ and did not find the second equation.

This question highlighted a need for candidates to practise more questions because they made too many errors in simplification. Also, there was evidence of a need for candidates to have more experience in recognising when the Chain or Product Rule would be most appropriate in differentiating a given expression. Also evident here was the need to remind candidates that numbers have both positive and negative square roots, and that a normal is perpendicular to a tangent, so that its gradient would be the negative reciprocal of that of the tangent (and vice versa).

Solutions

(a) $\frac{dy}{dx} = \frac{5x}{\sqrt{5x^2 - 4}}$

(b) $y = -\frac{1}{16}x + \frac{129}{16}$

(c) Equations are: $y = 8x - 16$ and $y = -8x - 16$

Question 6

This question tested candidates' ability to:

- Integrate a given trigonometric expression;
- Evaluate the definite integral of an algebraic expression;
- Determine the finite area bounded by a curve and two lines parallel to the $y$-axis;
- Determine the equation of a curve given its gradient function and a point on the curve.

The mean mark on this question was 7.34 with standard deviation of 4.78. The mean represented 52.4 per cent of the maximum possible 14 marks.

This question was generally relatively well done by candidates, particularly Part (a).

In Part (a) (i), most candidates knew how to integrate the trigonometric functions *sin* θ and *cos* θ; however, some neglected to include the constant of integration, C. In Part (ii), there were many incorrect attempts in the integration of $\frac{2}{x^2}$ and as a result candidates' answers were incorrect. Most candidates knew how to substitute the limits correctly to obtain an answer for their integration expression. There were a few candidates, however, who mixed up the substitution of the limits in that they used the lower limit minus the upper limit.

Part (b) was relatively well done by candidates. There were cases, though, where candidates included a constant of integration even though this question required the integration of a definite integral. There were a few errors with the arithmetic as well.

Part (c) was successfully completed by most candidates. The most common error seen here was a failure to include the constant of integration in the first instance of integrating the given function, and so not determining a value for it, and the concomitant errors therefrom. There were a few candidates who did not recognize that what was needed here was an integration of the given derivative to arrive at the original equation of the curve, and proceeded to differentiate the given derivative.

Teachers are asked to remind candidates of the need to include the constant of integration for integration of indefinite integrals and, as well, that this constant is not needed for integration of definite integrals. It seemed that many candidates were not clear on what the constant of integration meant. As well, the relationship between differentiation and integration as inverses of each other should be highlighted during teaching for candidates to acquire a better understanding and appreciation of the concepts.

**Solutions**

(a) (i) $3 \sin \theta + 5 \cos \theta + C$

(ii) $35\frac{1}{3}$

(b) $5\frac{1}{3}$ units$^2$

(c) Equation of curve is $y = x^3 - 3x^2 + 4$

**Section IV: Basic Mathematical Applications**

Question 7

This question tested candidates' ability to:

- Construct a stem and leaf diagram for a given set of data and (use it to) determine the mode, median and interquartile range for the data set;
- Use the laws of probability to determine the probability of the intersection and conditional probability of two events occurring, and to determine whether the events are independent;
- Determine the probability of drawing three balls of the same colour from a bag if replacements are allowed after each draw.

There were approximately 2351 responses to this question, making it the preferred choice of the two questions in this section. The mean mark was 10.63 with standard deviation 5.06. The mean represented 53.2 per cent of the total possible 20 marks.

This question was fairly well done with more than one-half of the candidates attempting it. Most of the candidates who attempted the question had some idea about at least one of the parts of the question with a fair number scoring more than 10 marks (half of the possible total). Candidates performed best on Parts (a) (i), (iii) and (iv) as well as (b) (i).

In Part (a), most of the candidates knew the general form of a stem and leaf diagram however, some did neglect to include a key. Some candidates also recognized the predominant advantage of the stem and leaf where this type of diagram retains the original information; some, however, were unable to clearly state an appropriate advantage of this form of data representation. The mode and median were also generally widely known. Finding the interquartile range, though, proved to be more difficult for candidates. A substantial proportion of candidates used the valid statistical formulas $\frac{1}{4}(n + 1)$ and $\frac{3}{4}(n + 1)$ for finding the lower and upper quartiles respectively, obtaining that these must be in the $6.25^{th}$ and $18.75^{th}$ positions. However, they then went on to round down and up respectively, taking the lower and upper quartiles to be the numbers in the $6^{th}$ and $19^{th}$ positions, without due consideration of the decimal values obtained with respect to the position of the quartiles – this, even though many of these candidates had used the formula $\frac{1}{2}(n + 1)$ for finding the position of the median value, and having obtained that this was in the $12.5^{th}$ position then correctly found the mean of the numbers in the $12^{th}$ and $13^{th}$ positions.

For Part (b) (i) most candidates could correctly determine $P(A \cap B)$ although some candidates used the formula $P(A).P(B)$ – which 'worked' only as the events were independent – rather than that given in the formula list at the front of the question paper. In Part (b) (ii) many candidates were not successful in determining $P(A|\ B)$. Candidates either did not know the formula, or even if they knew it, misapplied some aspect of it, for example inserting an incorrect value for $P(A \cap B)$ even in cases where they would have correctly worked this out previously. There were cases where candidates ended up with a value for probability greater than 1, seemingly not recognizing that this answer could not be correct. Part (b) (iii) was generally poorly done. Although many candidates could state that the events were independent, they could not give a valid mathematical reason, often simply stating that the events did not depend on each other to occur (that is, interpreted as a re-stating of the everyday definition of 'independent').

In Part (c) common errors generally had to do with candidates who added the probabilities for a single colour after each replacement (rather than multiplying them), or candidates who worked the problem without replacement. Also, there were candidates who correctly calculated the probabilities for each of the three colours separately, but then failed to add them together for the total probability. A few candidates opted to construct a probability tree diagram and generally were able to correctly solve the problem.

Whilst this question was relatively well done in the context of this year's paper, teachers are asked to take note of the identified weak areas above, and to assist candidates in recognizing, working through and strengthening these areas. One such weak area, for example, was related to independent events, and more work in teaching and learning could be done here.

Solutions

(a) (i)

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **1** | 5 | 6 | 8 | 8 | | | |
| **2** | 0 | 1 | 2 | 2 | 2 | 5 | 8 |
| **3** | 0 | 0 | 2 | 5 | | | |
| **4** | 0 | 1 | | | | | |
| **5** | 2 | 4 | 9 | | | | |
| **6** | 0 | 5 | 8 | | | | |
| **7** | 5 | | | | | | |

Key: 3 | 2 means 32

(ii) Advantage: Retains the original data set.

(iii) Mode is 22

(iv) Median is 30

(v) Interquartile range is 31.5

(b) (i) $P(A \cap B) = 0.4$

(ii) $P(A | B) = 0.5$

(iii) *A* and *B* are independent events as $P(A).P(B) = 0.4 = P(A | B)$

(c) $P(\text{3 balls same colour}) = 0.118$

<u>Question 8</u>

This question tested candidates' ability to:

- Draw a displacement-time graph from given information and use that graph to make requested calculations of speed and velocity;

- Determine the velocity of a particle given a description of its motion and an equation specifying its acceleration, and further determine its displacement over a given time period.

There were approximately 2010 responses to this question. The mean mark was 10.14 with standard deviation 5.65. The mean represented 50.7 per cent of the total possible 20 marks.

In Part (a) (i) for the graph, while many candidates were able to come up with reasonable scales and represent the first two parts of the journey correctly, only a relatively small percentage showed the negative displacement correctly. In fact, many candidates did not recognize that there were supposed to be negative values on the *y*-axis and hence left them out completely. In Part (ii) most candidates knew the formula for average speed; the problem here, though, tied in with Part (iii) as they could not distinguish between speed and velocity, and so appeared to not quite know what was being asked for between these two questions and had the same answers in both cases. Overall, Part (iii) was poorly done. Even in cases where candidates did correctly calculate the average velocity some of them ignored the minus sign, seemingly suggesting that negative velocity was not possible.

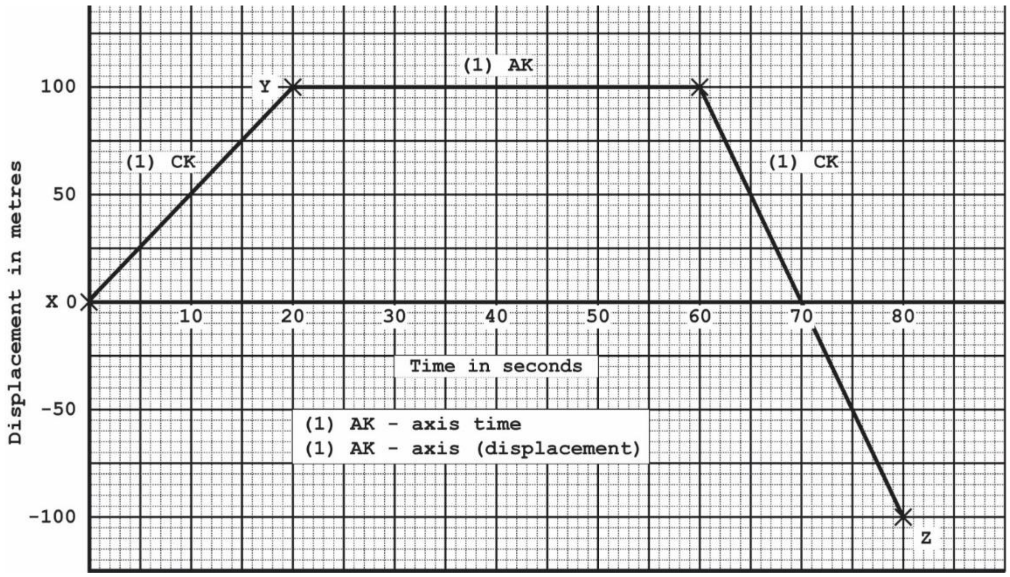
For Part (b), most knew that $v = \int a$ but there were a few who used $v = u = at$ to get $v$ and hence obtained an incorrect solution. As a result, because of their incorrect solution in Part (b) (i), the error was followed through in Part (ii) but they did understand the concept of $s = \int v$ between the limits $2\pi$ and $\pi$.

In Part (b) (i), a fair proportion of candidates knew they needed to integrate the given equation; some, though, had problems integrating the trigonometric function. Others, having integrated correctly, included a constant of integration in their answer, not recognizing the time interval outlined in the question for the particle's motion. For Part (ii) most of those who attempted this part after answering Part (i) correctly were also able to correctly complete it. A few, however, continued to include the constant of integration in their answer even though a definitive time interval was given in the question.

Teachers are asked to emphasize the differential relationship between displacement, velocity and acceleration. As well, it was evident that candidates needed to have more examples and practice of the idea of negative velocity, and a clearer understanding of the difference between speed and velocity. Appropriate use of the constant of integration is also an area for improvement.

**Solutions**

(a) (i)




(ii)

Average speed = 3.75 m/s

(iii) Average velocity = –1.25 m/s

(b) (i) *v = sin t*

(ii) displacement = −2

**Paper 031 — School Based Assessment (SBA)**

The marking team moderated approximately 150 centres by face-to-face marking, with a further 40 centres moderated via online marking. Generally, the projects were not of a very high quality. It was found that candidates researched similar topics compared to previous years. There were very few creative projects. The modal mark was 16, with mean of 16.10 and standard deviation 4.14. In 2015 the mean was 16.00 and standard deviation 3.17, highlighting that there was more variation in the SBA mark distribution this year. Few candidates obtained scores of 19 or 20, as in previous years. The number of Project A's and project B's was relatively even.

That said, there continued to be a number of observed weak areas in the samples submitted, related to the submissions themselves as well as to the marking of these. The following were general observations on both categories of projects:

- Some SBA project titles, aims and purposes were not clear, not concise and not well defined. Some titles were too vague, others too wordy;
- In far too many cases, candidates did not state the project's purpose. The purpose tells why the study is being conducted;
- A few submissions did not come with the required CSEC Additional Mathematics rubric;

- Findings were stated but few inferences were made concerning many of the stated findings which is one of the key goals of the SBA;
- A number of projects did not flow logically. Therefore, they were not awarded the two marks available for overall presentation;
- A few projects were incorrectly categorized and assessed, that is, a Project A being labelled and assessed as a Project B, and vice versa. This did create some problems as for example, Project B requires the collection and analysis of data from an experimental-type activity.

There was one instance where the project was done as group work and a single project was submitted bearing the names of all the candidates. Even where the project is done as a group, ALL aspects of the write-up must show each candidate's individual effort. Only the raw data collected must be the same.

**Comments specifically for Project A**:

**Mathematical Formulation**

- In stating how the project was going to be done most candidates correctly identified all the important elements of the problem and showed understanding of the relationship between the elements.
- There was a noted improvement in the level of difficulty associated with the projects. There was an attempt to attain the expected level for Additional Mathematics.

**The Problem Solution**

- Assumptions were often not clearly stated. Some candidates, though, did state limitations of the project.
- Explanations were generally not sufficient and also were not placed between every successive step in the problem solution. Candidates should devote more time to explanations and assume that the reader does not have context about the problem.
- Calculations were often precise but solutions were not clearly stated in a manner that was easy to follow and understand.

**Application of Solution**

- Many candidates were unaware of how to show that the solution or proof of the given problem was valid; the moderators needed to see how the candidate's solution could be applied elsewhere or to substitute their values to check for validity.

**Discussion of Findings/Conclusion**

- In some instances the discussion was worthwhile. However, the discussion was not always related to the project's purpose and was often quite broad.
- Conclusions in many instances were not found to be valid. Some candidates stated many conclusions which were not related to the purpose of the project.
- There were a few instances where suggestions for future analyses were stated or included in the project. Candidates did not identify how analyses/findings could be improved or used in related scenarios. It is recommended that more attention be given to this area.

**Comments specifically for Project B**:

**Method of Data Collection**

- Although candidates were not penalized, they should be encouraged to state specific types of data (primary, secondary, discrete, continuous) and types of sampling (random, stratified, quota) used in their projects.

**Presentation of Data**

- Too many candidates had the minimum required one table or chart; however, some tables and charts were not properly named or labelled. For example some graphs were produced using software programmes.
- There was an improvement in the systematic layout and organization of the tables and charts used by candidates.

**Mathematical Knowledge (Analysis of Data)**

- In this section, the candidate was expected to outline HOW the project was going to be done; this was not always included in the project.
- Mathematical concepts in a few cases were not used appropriately from the Additional Mathematics syllabus. Some candidates went beyond the scope of the syllabus.
- Most of the calculations shown were accurate. However, in some cases no working was seen because some candidates used software to generate the values.
- Generally some analysis was attempted, but the analysis was often not coherent. This could be attributed to the fact that no proper summary table of calculation values was seen, so the marker had to search through the document constantly to link the calculated values with the analysis.
- The candidates were not too clear on which two approaches were to be used in the analysis. In many cases the two approaches used were of the same concepts; for example, mean, mode and median were presented as different approaches to the analysis, but all these are measures of central tendency.

**Discussion of Findings/Conclusions**

- In most instances, there was no statement of findings and in some cases if it was in the project, it was not properly placed.
- Conclusions made were based on the reported findings but often were not related to the purpose of the project. As a result this made their conclusions invalid.
- In many cases there were no suggestions for future analysis. Candidates often mentioned the increase of sample size as a suggestion. However, suggestions for future analysis should show ways in which the project could be developed and/or used in a related area.

## Recommendations

The following recommendations hold for continued improvement of the SBA component of the Additional Mathematics examinations:

- All projects should have a clear and concise title, and well defined aim(s) or purpose;
- Where possible the SBA should identify with authentic real world situations;
- The variables that are being used (Project B) must be clearly stated and described. Variables can be controlled, manipulated and responding;
- The type of sample and sample size if relevant must be clearly stated. Some reasoning for why this sample size was chosen can be included;
- Teachers must ensure that projects which integrate other subject areas utilize concepts as contained in the CSEC Additional Mathematics syllabus;
- If candidates collect their data in a group setting, candidates MUST demonstrate their individual effort in relation to analysis (interpretation) and finding(s)/conclusion(s);
- Projects involving dice, coins, candy or playing cards must be more expansive so that candidates can present a more in-depth analysis of the topic under consideration. This can include looking at speciality dice, and additional card games that can allow candidates to demonstrate greater mathematical skills and abilities;
- As good practice, candidates should be encouraged to cite all sources and insert a reference/bibliography page;
- Teachers should guide candidates using the assessment criteria found in forms ‘Add Math 1- 5A’ and ‘Add Math 1-5B’ which are both available on the CXC website. Teachers can give their candidates the rubric as a means of guidance in developing their projects;
- Generally, it is clear that training workshops are needed in the area of project work for Additional Mathematics. Therefore, it is recommended that workshops be held throughout the Caribbean. It was apparent that some teachers had difficulties in understanding some aspects of the project. They first need to fully understand the requirements of the project to ensure that the content level in the project coincides with the Additional Mathematics syllabus.

**Paper 032 — Alternative to the SBA**

This question tested candidates' ability to:

- Model a real-world problem in mathematical form and so determine a value which minimized its solution (determine among three designs – cylindrical, rectangular and hexagonal – which one would minimize the amount of material needed for containers to hold a specific volume of paint).

There were 183 candidates who wrote this paper, of whom 114 attempted the question. The mean mark was 1.80 with a standard deviation of 3.66 compared with 5.21 and 4.53 respectively in 2015. The modal mark obtained by candidates who attempted the question was 0, with more than 40 per cent of candidates receiving this mark. More than 50 per cent of candidates received marks of 0 or 1. Five candidates received at least one-half of the total possible of 20 marks, with three candidates receiving full marks.

This paper was very poorly done. The fact that more than 40 per cent of candidates received zero means that a large proportion of candidates could not correctly complete Part (a) of the question which asked them to convert the stated volume of paint 19 litres to $m^3$ which is a relatively basic conversion that candidates are expected to know. Generally, candidates seemingly could not find a way 'in' to the question; that is, they could not begin to put the equations together in a step towards solving the problem.

There was a suggestion that perhaps consideration be given to breaking up the question on this paper to provide candidates with more scaffolding towards being able to solve it. The question on this year's paper had in fact already been broken up as much as was deemed possible to allow such; for example, with the inclusion in Part (b) of the desired end point (minimum surface area) for Design I and provision in Part (d) of the equation for area of Design III in order to allow candidates a starting point for determining the minimum surface area for that design.

**Solutions**

(a) 19 litres = 0.019 $m^3$

(b) Area (Design I) = $\frac{0.038}{r} + 2\pi r^2$ $r = 0.145m$ at minimum; substitution yields required result

(c) Minimum surface area (Design II) = 40.427 $m^3$

(d) Minimum surface area (Design III) = 0.407 $m^3$

(e) Design I is recommended as it uses the least material for containers to hold the given volume of paint.
Dimensions: radius = $0.145m$, height = $0.288m$

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

# REPORT ON CANDIDATES' WORK IN THE CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION

MAY/JUNE 2017

## ADDITIONAL MATHEMATICS
## GENERAL PROFICIENCY

## GENERAL COMMENTS

The 2017 setting of Additional Mathematics represented the sixth year the subject was being offered. This year also saw the continuation of the subject being marked on an e-marking platform, and included the SBA component for the first time.

The examination consisted of three papers:

- Paper 01 — 45 multiple choice questions
- Paper 02 — eight structured, essay questions
- Paper 031 or Paper 032 — Paper 031 is a School-Based Assessment (SBA) project component for candidates in approved educational institutions, whilst Paper 032 is an alternative to the SBA for out-of-school (private) candidates.

Questions in the examination tested the content and specific objectives of the Additional Mathematics syllabus with the aim of ensuring wide coverage of the syllabus. The questions were designed at the appropriate level to test the following skills: conceptual knowledge, algorithmic knowledge and reasoning.

Paper 01 tested content from Sections 1, 2 and 3 of the syllabus. Paper 02 tested content from all four sections of the syllabus. As before, the paper was structured around these four sections with each section containing two problem-solving questions. The questions in Sections 1, 2 and 3 were all compulsory. Section 4 consisted of two questions, one on Data Representation and Probability and the other on Kinematics. Each question was worth 20 marks. Candidates were required to answer only one question from this final section. Paper 031 is the SBA component of the examination. Candidates are required to do one project chosen from two project types: mathematical modelling (Project A) and data handling/statistical analysis (Project B). The SBA component is worth 20 marks. Private candidates can sit an alternate paper to the SBA, Paper 032, which consists of one in-depth, extended question from Sections 1, 2 and/or 3 of the syllabus. This paper is worth 20 marks.

This year, 4927 candidates registered for Additional Mathematics, which is an eight per cent increase over the number of registered candidates in 2016.

With respect to overall performance, 70 per cent of candidates earned Grades I–III, compared with 68 per cent in 2016.

## DETAILED COMMENTS

### Paper 01 — Multiple Choice

This was a 45 item paper covering Sections 1, 2 and 3 of the syllabus. The mean and standard deviation on this paper were 38.68 and 14.05 respectively (weighted up to a total of 60 marks), compared with 35.55 and 13.03 in 2016.

## Paper 02 — Structured Essay Questions

This paper consisted of eight questions, of which Questions 1 to 6 (Sections I–III) were compulsory. Candidates then had to choose either Question 7 or 8 (Section IV). The total possible score on this paper was 100 marks. The mean and standard deviation for this paper were 40.09 and 24.86 respectively, compared with 44.05 and 25.02 in 2016.

### Section 1: Algebra and Functions

Question 1

This question tested candidates' ability to

- determine the inverse of a given function, make use of the remainder and factor theorems to determine values for two unknown variables
- apply logarithms to reduce a given relationship to linear form and plot the graph to determine values for variables in that relationship.

The mean mark on this question was 5.21 with a standard deviation of 3.72. The mean represented 37.2 per cent of the possible 14 marks. Overall, candidates performed reasonably on this question. Parts (a) and (b) were fairly well done. This question required candidates to find the inverse of a given function and to use the remainder and factor theorems to find values for two unknown variables.

For Part (a), many candidates expressed the given function in terms of *y* and interchanged variables *x* and *y,* and displayed an inability to make *y* the subject of the formula to find the expression for the inverse of the function. At this level, it would be expected that this concept would be well handled by candidates.

In Part (b), some candidates had challenges formulating the equations to find the values of the two unknown variables. Others displayed some weakness in solving simultaneous equations. Generally, weaknesses displayed by candidates in Parts (a) and (b) of this question related to concepts which ideally they should have mastered from the CSEC General Mathematics syllabus.

Part (c) was the most challenging subpart of Question 1. Many candidates were unable to perform the reduction of the given equation to linear form. For those who did, many did not use a reasonable scale to plot the graph and hence could not compute a value for the constant *A* accurately. Still others did not recognize that they needed to read the value of the *y*-intercept off their graph to find the value of *A* — the 'hence' aspect of Part (c) (iii). Notably, there were a few candidates who used natural logarithms to solve this question.

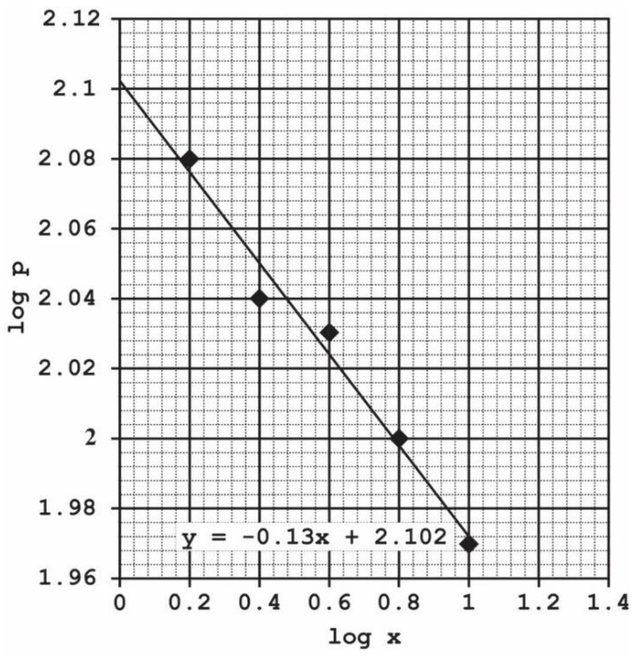
Parts of this question highlighted that some candidates sitting Additional Mathematics are still weak in some areas which could be considered basic content in the CSEC General Mathematics syllabus. Teachers are asked to give some attention to concepts such as changing the subject of formulae and solving equations, in particular simultaneous equations. More work also needs to be done with candidates on choosing appropriate scales for graphs based on the data presented to them.

**Solutions**

(a) (i) $f^{-1}(x) = \frac{p + x}{x - 2}$ (ii) $p = 22$

(b) $a = 4, b = 4$

(c) (i) $\log P = \log A - k \log x$ (ii)




(iii) $A = 126.5$; $k = 0.13$

Question 2

This question tested candidates' ability to

- make use of the relationship between the sum and product of the roots of a quadratic equation
- find the solution set of a rational inequality with linear factors
- convert a real-world problem to a solvable mathematical problem given that the problem follows a geometric series (that is, obtain expressions for the general terms and sum of a geometric series in order to solve a real-world problem).

The mean mark on this question was 5.81 and the standard deviation was 4.32. The mean represented 41.5 per cent of the possible 14 marks.

Overall, candidates performed relatively well on this question with Part (b) posing the most difficulty.

In Part (a), candidates were generally successful in obtaining the sum and product of the roots of the given quadratic and used them correctly as required by the question. A minority were unable to do so successfully. For Part (b), which proved the most difficult part of Question 2, many candidates had challenges correctly stating the solution range. Algebraic methods were the most common routes used for solving; however, there were too many candidates who failed to multiply by the square of the linear denominator factor. Further, in stating the solution range, too many candidates again failed to recognize that $x \neq -1$, so that $x \geq -1$ was included in the solution set.

Both Parts (c) (i) and (ii) were generally well done by candidates; many used their geometric progression knowledge to solve the problem. Nonetheless, there were several candidates who attempted to use an arithmetic progression method to solve, even though the question stated that the increase followed a geometric series.

Teachers are to be commended for the obvious work done with getting students to understand and be able to make use of the ideas of the sum and product of quadratic equations. Like 2016, it was recommended that students needed more exposure to and practice in translating real-world word problems into solvable mathematical forms, especially as it relates to the use of arithmetic progression and geometric progression. There was noted improvement in performance on this question type this year. However, more work needs to be done in the area of solving inequalities, in particular as it relates to correctly stating the solution set.

**Solutions**

(a) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{-6}{7}$

(b) Range of values: $x\{x > -1$ **or** $x \leq \frac{-3}{2}\}$

(c) (i) $48 044.50 (ii) $48 044.50 (some allowances made here for rounding)

**Section II: Coordinate Geometry, Vectors and Trigonometry**

<u>Question 3</u>

This question tested candidates' ability to do the following:

- Write the equation of a circle in the form $(x + f)^2 + (y + g)^2 = r^2$. Use that form to state the coordinates of the centre and value for the radius of the circle, and determine the coordinates of the points of intersection of the circle with a straight line segment
- Find the equation of a tangent to the circle at a particular point, given the coordinates of the end points of a diameter.
- Determine the scalar product of two vectors and find the angle between them.

The mean mark on this question was 5.00 and the standard deviation 3.98. The mean represented 41.7 per cent of the possible 12 marks.

This question was reasonably well done, with more than 40 per cent of candidates who attempted it scoring more than six marks and 14 per cent scoring 11 or 12 marks. However, some candidates displayed no knowledge of this topic and either omitted the question or failed to score any marks. In fact, about 35 per cent scored two or fewer than two marks.

Areas of good performance were seen in Parts (a) (iii) and (b) (ii), whilst candidates displayed weaknesses in Parts (a) (i), (ii) and (b) (i).

For Part (a) (i), candidates were unable to correctly complete the square on the terms in *x* and the terms in *y*, so that $(x + 4)^2 + (y - 2)^2 = 20^2$ was often seen. Even for those who completed the squares correctly, they invariably wrote their equation as $(x + 2)^2 + (y - 1)^2 = 25$. In Part (a) (ii), though some candidates correctly wrote the coordinates of the centre of the circle and the value of the radius from their form of the equation of the circle, many tried using formulae to calculate the coordinates and the value of the radius and did so incorrectly. A few candidates also had a zero radius, a negative radius, or the radius as the square root of a negative number which indicated poor understanding of the general forms of the equation of a circle. Obtaining a zero radius or the square root of a negative number should have raised a red flag for candidates, indicating that something was wrong with their calculations. Part (a) (iii) was better handled by candidates as they attempted to substitute both places for *y* into the equation of the circle and were generally able to expand correctly. They were also generally able to solve the resulting quadratic equation correctly for *x*, even if the equation was not the correct one, and as a result were able to solve for two values of *y*.

Part (b) (i) presented some difficulties for candidates. The use of the phrase find *the product of the vectors* seemingly caused confusion for some candidates and answers such as $2i^2 + 13ij + 15j^2$ or $2i + 15j$ were frequently seen. Many candidates were, however, able to calculate ***p*.*q*** when using the formula to find the angle between the vectors. Part (b) (ii) was better handled; candidates were generally able to find the correct length of at least one vector and to follow through to find the angle $\Theta$ made by the two vectors. A few candidates treated the two vectors as position vectors and were able to correctly find the angle each made with the horizontal axis and then subtracted to find the angle between the vectors.

**Recommendations**

The following recommendations are for teachers:

- Students need to be given more examples which involve finding the centre and radius of a circle when written in its two general forms. Reinforcement of the technique of completing the square should be a priority. Emphasis should also be placed on deriving the formulae rather than memorizing them.
- Teachers should also point out to students that the radius of a circle cannot be the square root of a negative number or zero.

- More work and practice needs to be done on the use of the scalar/dot product to find the angle between two vectors.

**Solutions**

(a) (i) Circle equation: $(x + 2)^2 + (y - 1)^2 = 5^2$ (ii) Centre (–2, 1), Radius = 5

(iii) Points of intersection (−2, 6) and (3, 1)

(b) (i) $\mathbf{p.q} = (2 \times 1) + (3 \times 5) = 17$ (ii) $\theta = 22.4°$ or 0.391 rads

Question 4

This question tested candidate's ability to

- apply knowledge of the formula for area of a sector of a circle to solve a problem involving a compound-shape
- make use of the compound/double angle formulae and to use the exact rational values for sine, cosine and tangent of particular angles to simplify a given trigonometric fraction in surd form
- prove a trigonometric identity

The mean mark on this question was 3.93 and the standard deviation 3.94. The mean represented 32.9 per cent of the possible 12 marks.

Part (a) was best done by candidates; Parts (b) and (c) presented difficulties for candidates, some of which related to candidates still having a poor grasp of the concept of fractions, and what is and is not possible when simplifying these – that is, basic concepts which should have been mastered by this stage.

There was a noted ambiguity in Part (a) of the question caused by the use of 'section ABC' for what should have been 'sector ABC'. This did mean that some candidates may have treated DCB as a sector as opposed to the intended ABC. Nonetheless, in Part (a) (i), most candidates were able to obtain the one mark allotted for determining the size of angle ACB. In Part (a) (ii), having determined the size of angle ACB, candidates were generally able to go on to determine the size of angle ACD and at least suggest a method for determining the area of triangle ACD (the area used for farming). A noted weakness though for Part (a) related to some candidates working and giving their answers in degrees even though the question had asked for this in radians.

Part (b) was generally poorly completed. Most candidates recognized the need to use the compound angle formula for the numerator and were able to substitute accordingly. Some used the subtracted form of the formula in the expansion, even though this formula was provided on the formula sheet. For the denominator, candidates who successfully completed the substitution here either recognized it as a double angle and worked to suit, or recalled that sin $\frac{2\pi}{3}$ = sin $\frac{\pi}{3}$ or else they split $\frac{2\pi}{3}$ into $\frac{\pi}{3}$+ $\frac{\pi}{3}$ and used the compound angle formula. However, some candidates appeared to have problems manipulating the surds and hence ran into difficulty arriving at the target final expression. Many simply took a leap and concluded their workings there, with no logical follow through to the final step.

In Part (c), many candidates only got as far as a statement of the required substitution to begin the proof, that is, a version of $\cos^2\theta + \sin^2\theta = 1$. Others, having made this statement, proceeded to substitute incorrectly, or failed to manipulate the simplifications correctly, many of the errors having to do with simplifying fractions. For example, many candidates were cancelling *sin ϑ* across the addition sign in the numerator and denominator. Focus should be placed on working with fractions (determining the LCM, when to cancel) in addition to working with directed numbers.

Teachers should ensure that students can accurately perform basic arithmetic operations with numbers first before attempting similar operations with trigonometric proofs. They perhaps need to remind students that even though they are learning new methods in Additional Mathematics they should not forget the basic techniques of Mathematics. Additionally, when asked to prove or show something in Mathematics, it is important to show the processes to the target in step by step manipulations, and not to suddenly arrive at the target from a leap too far.

**Solutions**

(a) (i) $\theta = \frac{\pi}{3}$ radians (ii) Area of $\Delta$ ADC = $\frac{25\sqrt{3}}{2}$ $m^2$

(b) $$\frac{\cos(\frac{\pi}{4}-\frac{\pi}{3})}{\sin\frac{2\pi}{3}} = \frac{\cos\frac{\pi}{4}\cos\frac{\pi}{3}+\sin\frac{\pi}{4}\sin\frac{\pi}{3}}{2\sin\frac{\pi}{3}\cos\frac{\pi}{3}}$$

$$= \frac{\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)+\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)}{\frac{\sqrt{3}}{2}} = \frac{\frac{\sqrt{2}}{4}+\frac{\sqrt{2\times 3}}{4}}{\frac{\sqrt{3}}{2}} = \frac{\sqrt{2}+\sqrt{6}}{2\sqrt{3}}$$

(c) $$1 - \frac{\cos^2\vartheta}{1+\sin\vartheta} = 1 - \frac{1-\sin^2\vartheta}{1+\sin\theta} = 1 - \left(\frac{(1-\sin\vartheta)(1+\sin\theta)}{(1+\sin\theta)}\right) = 1-(1-\sin\vartheta) = \sin\vartheta$$

**Section III: Introductory Calculus**

Question 5

This question tested candidate's ability to

- differentiate a given algebraic polynomial and simplify their answer
- determine the equation of a normal to a curve at a point on the curve
- use the concept of derivative to determine the rate of change of the volume of water being poured into a cylinder.

The mean mark on this question was 5.18 and the standard deviation 4.18. The mean represented 37 per cent of the possible 14 marks.

Part (b) was done best by candidates. Parts (a) and (c) presented particular difficulties for a majority of candidates.

In Part (a), candidates generally knew they needed to apply the product or the chain rule or both, or alternatively that they could expand the expression and then differentiate. However, in either approach, candidates generally encountered difficulties. In the first approach, candidates displayed weaknesses in obtaining the two terms after differentiating. Some differentiated each term correctly, but seemingly forgot the aspect of holding one term constant while differentiating the other term and multiplying. Where they did so, for the most part, they obtained the $(1 + 2x)^3 \times 1$ term correct, but many found difficulty using the chain rule in order to differentiate the $(1 + 2x)^3$ term, in that they forgot (or did not know) to also differentiate $2x$ then multiply in order to get $6(1+ 2x)^2$. Having moved beyond this step, some candidates still had difficulty factorizing the expressions and others did not simplify correctly. There were a few cases of candidates attempting to apply the quotient rule to this differentiation. In the second approach, candidates attempted to expand then differentiate; many candidates ran into difficulties in correctly expanding the given algebraic expression.

In Part (b), for the most part, candidates knew how to differentiate the given equation for the curve, and did so correctly. There were, however, a few candidates who integrated the given curve equation. There were a number of candidates who, on finding the gradient value at the given point, used that to determine an equation for a tangent, rather than finding the gradient of the normal at the point. Those who did find the gradient of the normal knew it to be the negative reciprocal of the gradient of the tangent, and were most often able to continue and correctly determine the equation of the normal as required.

In Part (c), although a fair number of candidates knew they needed to differentiate the volume of cylinder formula, they did so with respect to the radius, $r$, and not with respect to the height, $h$, which in the context of the question was the variable (in terms of height of the water in the cylinder) that was changing. As a result, they ended up with a wrong determination of the rate of increase of volume in terms of $\pi$. Some candidates were aware of the need to apply the chain rule in order to determine the rate of change of volume, but its set up to obtain the correct $\frac{dV}{dt}$ proved to be a challenge for many of them.

**Recommendations**

Students must be given enough practice exercises involving products, and products involving composite functions. Students seem to struggle with identifying products when differentiating, it is suggested that they be given examples and non-examples of products. They should also be able to distinguish among products, quotients and composite functions and the associated rules for differentiation as well as mixtures of these functions. On a more basic level, more time and practice should be given to the expansion, simplification and factorization of more complex algebraic expressions as students displayed particular difficulties. Rates of change is an application of differentiation, and time needs to be devoted to providing students with a conceptual understanding of this area so that they appreciate what is being asked of them in such questions.

**Solutions**

(a) $\frac{dy}{dx} = (1 + 2x)^2 (8x + 19)$

b) $y = -\frac{1}{4} x - \frac{1}{4}$

(c) $450 \pi$ $cm^3 s^{-1}$

Question 6

This question tested candidate's ability to:

- compute the definite integral of a trigonometric function
- formulate the equation of a curve given its gradient function and a point on the curve
- evaluate the definite integral of an algebraic function
- determine the volume of the solid formed when an area, in the first quadrant within given boundaries, is rotated about the *x*-axis.

The mean mark on this question was 5.58 and the standard deviation 4.88. The mean represented 39.9 per cent of the possible 14 marks.

This question was reasonably well done. More than 40 per cent of candidates who attempted it scored more than seven marks. Part (d) posed the most difficulty for candidates. Most candidates with a strong grasp of integral calculus performed well on this question, however, a relatively high percentage (27 per cent) displayed no knowledge of this topic evidenced by them either omitting the question or not scoring any marks.

In Part (a), the majority of candidates who integrated the trigonometric functions correctly knew the exact values for the sine and cosine of the angles $\frac{\pi}{4}$ and 0 radians and was able to arrive at the required result. Others who attempted this part either integrated incorrectly or were unable to produce the exact values for sine and cosine of the required angles because in some instances it appears they forgot to change the angle mode on their calculators from degrees to radians.

Part (b) was well done by candidates who recognized that they needed to integrate $\frac{dy}{dx}$ in order to find *y*. A significant number of them, however, only had $\int x+2$ and gave their answer as $x^2 + 2x$. Other candidates who correctly integrated $\frac{dy}{dx}$ and obtained $y = \frac{x^2}{2} + 2x + c$, then found the correct value of the constant, and continued to determine the equation of a tangent. Thus, $y = 4x - 3$ was often seen as the final answer.

In Part (c), integration performed by directly considering the given function often resulted in the candidate mistakenly dividing by 3 instead of −3. Some candidates also had problems with substituting the limits, and so $\int_1^2 (4-x)^2 = \left[\frac{(4-x)^3}{-3}\right]_1^2$ produced $\frac{4-2^3}{-3} - \frac{4-1^3}{-3}$. Many candidates opted to expand the given function before integrating. However, some (about one third) of these candidates were unsuccessful at squaring, $16-8x-x^2$ or $8-8x+x^2$ were frequently seen.

In Part (d), recalling the formula for the volume of the solid of revolution posed difficulty for the candidates. Some candidates recognized the solid of revolution formed as a cone but very few were able to recall and utilize the formula for the volume of a cone correctly. The following were some of the incorrect expressions seen:

$$V = 2\pi\int_0^6 \frac{x}{2}\,dx = 2\pi\left[\frac{x^2}{4}\right]_0^6 = 2\pi\left[\frac{6^2}{4} - \frac{0^2}{4}\right] = 18\pi$$

$$V = \pi\int_0^6 \frac{x}{2}\,dx = \pi\left[\frac{x^2}{4}\right]_0^6 = \pi\left[\frac{6^2}{4} - \frac{0^2}{4}\right] = 9\pi$$

$$V = 2\pi\int_0^6 \left(\frac{x}{2}\right)^2 dx = 2\pi\left[\frac{x^2}{4}\right]_0^6 = 2\pi\left[\frac{6^2}{4} - \frac{0^2}{4}\right] = 18\pi \quad \text{no integration}$$

$$V = \pi\int_0^6 \left(\frac{x}{2}\right)^2 dx = 2\pi\int_0^6 \frac{x^2}{4}\,dx = 2\pi\left[\frac{x^2}{4}\right]_0^6 = 18\pi \quad \text{no integration}$$

$$V = \pi\int_0^6 \left(\frac{x}{2}\right)^2 dx = \pi\left[\frac{x^2}{4}\right]_0^6 = \pi\left[\frac{6^2}{4} - \frac{0^2}{4}\right] = 9\pi \quad \text{no integration}$$

$$V = \int_0^6 \frac{x}{2} dx = \left[\frac{x^2}{4}\right]_0^6 = \left[\frac{6^2}{4} - \frac{0^2}{4}\right] = 9$$

**Recommendations**

Teachers need to insist that once integration has occurred, the integral sign and the *dx* should be dropped since too many candidates left the integral sign after performing either the definite or indefinite integral. Students should be encouraged to memorize the sines, cosines and tangents of the basic angles measured in degrees or radians, expressed in decimal or surd form. Students must also be given more examples in the use of integrating formula for composite functions such as $(b + ax)^n$ where $a$ is negative. Above all when doing calculus involving trigonometric functions, teachers must remind students to change the mode of their calculators from degrees to radians if they are to obtain correct results.

**Solutions**

(a) $\int_0^{\frac{\pi}{4}} (\sin x + 4\cos x)\, dx = [-\cos x + 4\sin x]_0^{\frac{\pi}{4}}$

$$= -\frac{\sqrt{2}}{2} + 4\left(\frac{\sqrt{2}}{2}\right) + 1 = \frac{3\sqrt{2}}{2} + 1 = \frac{3\sqrt{2}+2}{2}$$

(b) Equation of curve is $y = \frac{x^2}{2} + 2x - 3$

(c) $\frac{19}{3}$

(d) $18\pi$ units$^3$

**Section IV: Basic Mathematical Applications**

<u>Question 7</u>

This question tested candidates' ability to

- Construct and use a tree diagram to solve problems involving probability
- Determine the mean, mode, median and interquartile range for a data set
- Determine the probability of events using the basic laws of probability.

There were approximately 2859 responses to this question, making it the preferred choice of the two questions in this section. The mean mark was 11.31 and the standard deviation 5.09. The mean represented 56.6 per cent of the possible 20 marks. This was the only question on Paper 02 to achieve a mean which was more than one-half of the total allotted marks.

This question was fairly well done with more than one-half of the candidates attempting it. Most of the candidates who attempted the question had some knowledge of at least one of the parts of the question. Candidates performed best on Parts (a) (i); (b) (i), (ii), (iii) and (c) (i).

In Part (a) (i), most candidates correctly placed the information given on a probability tree diagram. However, there were some careless mistakes in transposing the information given and summing the branches to total to 1, especially related to the branch for non-replies from firm *Q,* where 0.5 was sometimes seen instead of 0.05. There were some candidates who appeared to not know how a tree diagram should look, and tables or flow charts with arrows were sometimes seen for a probability tree diagram. Part (a) (ii) presented a challenge for some candidates. Whereas most were able to add the probabilities correctly, some just added the probabilities $0.95 + 0.3 + 0.3 = 1.55$ and placed that as their answer.

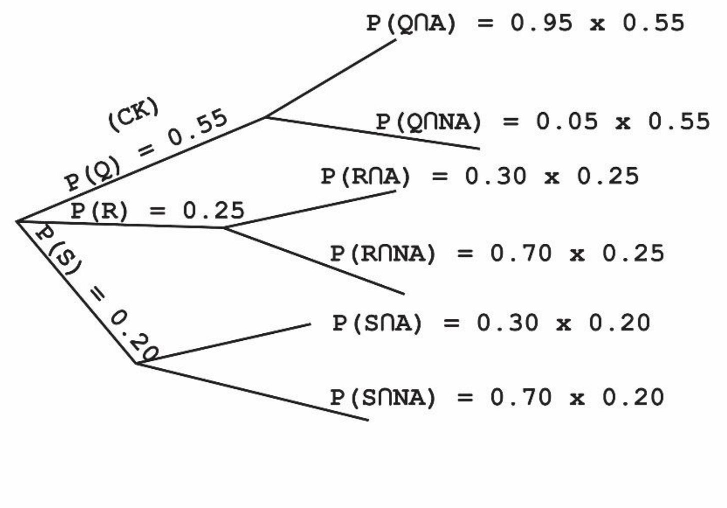
Parts (b) (i), (ii) and (iii) were generally well done by candidates, with mainly arithmetic errors. In Part (b) (iv), however, determination of the interquartile range posed a challenge for some candidates. Although most candidates knew the formula $IQR = Q_3 - Q_1$, determination of the quartiles was not always done correctly which resulted in incorrect final answers.

In Part (c) (i), most candidates were able to arrive at the probability of the first patty selected being saltfish, this was often given as $\frac{25}{55}$, without giving consideration to what the second patty might be. Only a few candidates attempted in their response to pick the second patty. In Part (c) (ii), some candidates were also able to determine the probability of the second patty not being saltfish. Candidates generally lost marks for not remembering that the sample space would now be one less given that the first patty had already been selected.

Candidate performance on this question is an indication that teachers have done a lot of good work in preparing students in this area. However, there were some areas of weakness. Teachers should ensure that students understand that on a tree diagram the sum of the probabilities on each branch must be one, that the probability of an event cannot exceed 1 (and the why – the conceptual thinking behind this). More work is also needed on the calculation of quartiles in determining interquartile ranges.

**Solutions**

(a) (i)




(ii) *P* (Student accepted) = 0.6575

(b) (i) Mean length = 21.735 cm
(ii) Modal length = 25.2 cm
(iii) Median length = 24.8 cm
(iv) Interquartile range = 17.95 cm

(c) (i) *P*(First of two is saltfish) = $\frac{25}{55}$

(ii) Given first is saftfish, *P*(Second NOT saltfish) = $\frac{30}{54}$

Question 8

This question tested candidates' ability to

- apply the rate of change of displacement of the motion of a particle from a given equation to make determinations related to its velocity, etc.
- draw a velocity–time graph from given information and use that graph to make other related calculations.

There were approximately 1763 responses to this question. The mean mark was 7.61 and the standard deviation 3.74. The mean represented 38.1 per cent of the possible 20 marks.

Generally, candidates performed well on both Parts (a) and (b) (i) but encountered great difficulty in finding the relevant time in Part (b) (ii), and also had difficulty with Parts (b) (iii) and (iv).

In Part (a) (i), generally, candidates recognized that they needed to differentiate the given expression for displacement with respect to time in order to determine velocity, and were able to do so. However, there were a few candidates who integrated instead of using differentiation. Others substituted the value for *t* into the expression given for *s*, thus finding a value for displacement rather than for velocity. In Part (a) (ii), a majority of candidates recognized that they needed to set the velocity, $\frac{ds}{dt} = 0$; however, some candidates set the given displacement equation itself, $s = 0$.

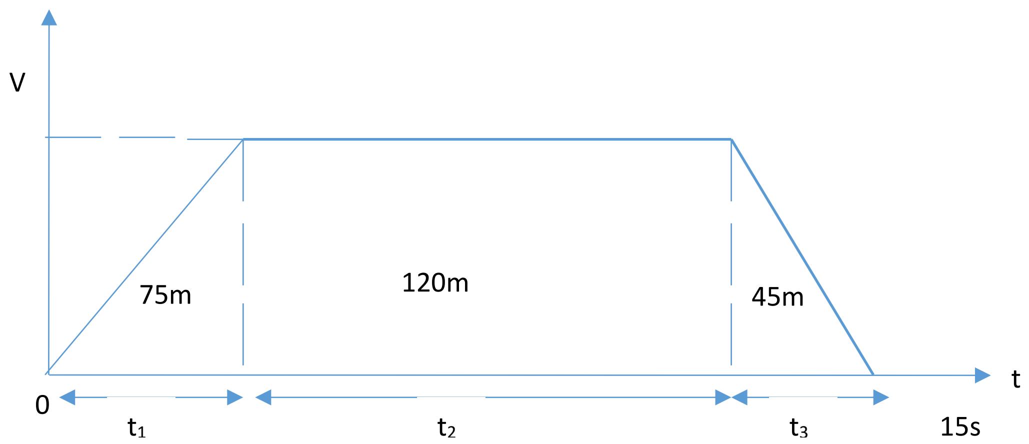
In Part (b), although generally a correct velocity–time graph generated was correct, in most cases, candidates were unable to find the required time for constant speed in Part (b) (ii). Candidates did not always recognize that in order to find the required length of time at constant speed, they needed to find the area under the graph; that is, the total distance travelled, and from that determine the length of time at constant speed. Failure to attempt Part (b) (ii) frequently resulted in candidates omitting Parts (b) (iii) and (iv) as well. The values used for the times in Parts (b) (iii) and (iv) also indicated that some candidates did not interpret what was being asked correctly. While some candidates were able to score full marks, this question posed great difficulty to the majority candidates who attempted it.

Candidate errors on this question highlighted particular misconceptions or misinterpretations of the syllabus content in this area. Teachers are asked to ensure students get more and varied practice in these types of questions as they were generally poorly done.

**Solutions**

(a) (i) Velocity = 17.25 $ms^{-1}$ (ii) Time = 2 seconds

(b) (i)




(ii) Time for constant speed = 5 seconds

(iii) $V_{max} = 24\ ms^{-1}$

## Paper 031 — School Based Assessment (SBA)

The marking team moderated approximately 890 projects. All projects were marked online. Generally, there was some improvement in the quality of the projects. It was found that candidates continued to research similar topics when compared to previous years. There were very few creative projects. The modal mark was found to be 16. Many projects did not score 19 or 20, as in previous years. The number of Project submitted for both A and B was relatively even.

There continued to be a number of observed weak areas in the samples submitted, related to both the submissions themselves as well as to the marking of these. The following were general observations on both Projects A and B:

- Some SBA project titles, aims and purposes were not clear, not concise and not well defined. Some titles were too vague, while others were too wordy.
- In far too many cases, students did not state the project's purpose. The purpose tells why the study is being conducted.
- A few submissions did not come with the required CSEC Additional Mathematics rubric.
- Findings were stated but little inferences were made concerning many of the stated findings, a key goal of the SBA.
- A few projects were incorrectly categorized and assessed, that is, a Project A being labelled and assessed as a Project B, and vice versa. This created some problems as, for example, Project B requires the collection and analysis of data from an experimental-type activity.

**Comments Specifically for Project A**

Title

- Candidates continue to name the project rather than state *what* the project is about.

Mathematical Formulation

- In stating the process for the project, most students correctly identified all the important elements of the problem and showed understanding of the relationship between the elements.
- There was a noted improvement in the level of difficulty associated with the projects. There was an attempt to attain the level of Additional Mathematics.

The Problem Solution

- Assumptions were often not clearly stated. Some students though did state limitations of the project.
- Explanations were generally not sufficient and also were not placed between every successive step in the problem solution.
- Calculations were often precise but solutions were not clearly stated.

Application of Solution

- Many students were unaware of how to show that the solution or proof of the given problem was valid. Students were required to show how their solution can be applied elsewhere or to substitute their values to check for validity.

Discussion of Findings/Conclusion

- In some instances the discussion was worthwhile. However, the discussion was not always related to the project's purpose and was often quite broad.
- Conclusions in many instances were not found to be valid. Some students stated many conclusions which were not related to the purpose of the project.
- There were a few instances where suggestions for future analyses were stated or included in the project. Students did not identify how analyses/findings could be improved or used in related scenarios. It is recommended that more attention be given to this area.

**Comments Specifically for Project B**

Method of Data Collection

- Students should be encouraged to state the specific types of data (primary, secondary, discrete, continuous) and types of sampling (random, stratified, quota) used in their projects.

Presentation of Data

- There was noted improvement in the systemic layout and organization of the tables and charts used by students. The majority of students achieved full marks in this section.

Mathematical Knowledge (Analysis of Data)

- In this section, students were expected to outline *how* the project was going to be done; this was not always included in the project.
- Mathematical concepts, in a few cases were not used appropriately from the Additional Mathematics syllabus. Some students went beyond the scope of the syllabus.
- Generally, some analysis was attempted, but the analysis was often not coherent. This could be attributed to the fact that no proper summary table of calculation values was seen; therefore, markers had to search through the document constantly to link the calculated values with the analysis.
- Students were not very clear on which two approaches were to be used in the analysis. In many cases, the two approaches used were of the same concepts, for example, mean, mode and median were presented as different approaches to the analysis, but all these are measures of central tendency.

Discussion of Findings/Conclusions

- In most instances there was no statement of findings. However, when the statement of findings was provided, it was not properly placed.
- Conclusions made were based on the reported findings but often were not related to the purpose of the project. As a result this made the conclusions invalid.
- In many cases there were no suggestions for future analysis. Students often mentioned an increase in sample size as a suggestion. However, suggestions for future analysis should show ways in which the project could be developed and/or used in a related area.

**Recommendations**

The following recommendations hold for continued improvement of the SBA component of the Additional Mathematics examination:

- All projects should have a clear and concise title, and well-defined.
- Where possible, the SBA should identify with authentic, real-world situations.
- The variables that are being used in Project B must be clearly stated and described. Variables can be controlled, manipulated and responding.
- The type of sample and sample size, if relevant, must be clearly stated. Some reasoning for why this sample size was chosen can be included.
- Teachers must ensure that projects which integrate other subject areas utilize concepts as contained in the CSEC Additional Mathematics syllabus.
- Projects involving dice, coins, candy or playing cards must be more expansive so that students can present a more in-depth analysis of the topic under consideration.
- As good practice, students should be encouraged to cite all sources and insert a reference/bibliography page.
- Teachers should guide students using the assessment criteria found on forms "Add Math 1-5A" and "Add Math 1-5B" which are both available on the CXC website. Teachers can give their students the rubric as a means of guidance in developing their projects.

**Paper 032 — Alternative to SBA**

This question tested candidates' ability to:

- model a real-world problem in mathematical form and so determine a value which maximized its solution (determine among two window designs (rectangular and compound-shaped (semi-circle and rectangular))
- determine which of the window designs would maximize the area for a given length of material to allow more light through the two window designs.

There were 183 candidates who wrote this paper of whom 150 attempted the question. The mean mark was 5.81 and the standard deviation 3.79, compared with 1.80 and 3.66 respectively in 2016.

Although there was some noted improvement compared with 2016, this paper was generally poorly done. Candidates performed best on Parts (a) and (c), but encountered great difficulty in responding appropriately to Part (b).

In Part (a), candidates were generally able to get two or three of the allotted eight marks. Candidates were able to at least suggest the variables for the area and/or perimeter of the two shapes given for window designs, and also to write expressions representing their area and/or perimeter. However, very few candidates scored full marks. Common errors included considerations of area only and not perimeter or vice versa, setting the area expression equal to 4 m rather than the expression for perimeter. For those candidates who did formulate the mathematical problem correctly with respect to the required equations, only a few went on to state the need to maximize the area subject to the given constraint of the perimeter being 4 m.

Part (b) posed great difficulty for a majority of candidates. Generally, candidates did not make the necessary substitutions to have the area of the designs expressed in terms of one of the variables, nor did they make attempts to find stationary values (that is, did not differentiate at all, or, if they did, did not set $\frac{dA}{dx} = 0$). For candidates who did set their differentiated expressions equal to 0, very few went on to confirm that the stationary point was indeed a maximum.

For Part (c), candidates were generally able to cite the correct window design that would maximize light, and also were able to attribute the reason to a larger area coverage—this outside of having worked Part (b) correctly. Candidates did seem to know that shapes that incorporated curvature, all other things being equal, would maximize area.

This year's Paper 032 question was a classical maximize area-type question, and a real-world application of the idea of derivative and stationary points. Especially as evidenced in Part (b), far too many candidates were unsure about their approach to answer this question. It is recommended that teachers spend more time in classes providing opportunity for problem-solving of real-world applications of mathematical concepts.

**Solutions**

(a) Dependent on variables used:

For Design 1, Maximize $A = \frac{\pi r^2}{2} + 2ry$, subject to $\pi r + 2y + 2r = 4$, where width = $2r$ and length = $y$

For Design 2, Maximize $A = xy$, subject to $2(x + y) = 4$, where width = $x$ and length = $y$

(b) Maximum Area: Design I = 1.12 $m^2$; Design 2 = 1 $m^2$

(c) Recommendation: Design 1, larger area so allows in more light